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FOUR HYDROPHONE ARRAY FOR ACOUSTIC THREE-DIMENSIONAL LOCATION

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13. ABSTRACT

This report describes a non-rigid, floating, four-hydrophone array that can give accurate three-dimensional locations for certain underwater sound sources, particularly for those that are close. For more distant sounds, the array can indicate good directions (bearings) but range is liable to be uncertain. A system for concurrent calibration permits periodic reassessment of hydrophone positions.

Appendix: Programming and mathematics for computer analysis of the array data are by Donna Ekstrand and Mary Hunt.

14. KEY WORDS	LINK A		LINK B		LINK C	
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1. Array						
2. Three-dimensional locations						
3. Source (sound) location						
4. Ambient noise						

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WOODS HOLE OCEANOGRAPHIC INSTITUTION
Woods Hole, Massachusetts

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FOUR HYDROPHONE ARRAY FOR ACOUSTIC
THREE-DIMENSIONAL LOCATION

William A. Watkins and William E. Schevill

October 1971

Details of illustrations in
this document may be better
studied on microfiche

TECHNICAL REPORT

Approved for Distribution ⟩R⋅Heintzle _____
J. R. Heirtzler, Chairman
Department of Geology & Geophysics

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Illustrations:

Introduction

One of the most frequent questions asked us about underwater animal sounds is, "How loud was it really?" As so often happens, the answer is nowhere near as simple as the question. Only if one knows the distance from source to receiver can the answer even be approached, and, as will appear in this report, an accurate measurement of this distance is not easy. Certain other factors are needed, too, such as the propagation characteristics of the water, and proper calibration of the receiver.

We are pretty well restricted to acoustics for underwater measurements and observations at any distance greater than a few meters, for of our available sensory paths only that of sound passes well through water. We cannot see well enough through water to use any optical means, and moreover, aquatic sound-makers are not readily identified by looking, even when visible (unlike most animals living in air, they do not open their mouths to phonate). The animal in sight (as in figure 1) may not be the only one producing the sound.

For many years we put this problem aside, because it seemed clear that it could be attacked only with an array, and an array of meaningful size promised to be a great nuisance to handle at sea.

Directional listening resolves some of the uncertainties by giving an indication of the bearing of the sound, thus reducing the choice of sound-producers to those in that direction. These listening systems include hydrophones and multiple units arranged to produce differences in intensity or sound-arrival-time with direction (see Cummings 1968). A 2-sensor array gives ambiguous bearings, but arrival-time-difference measurements of a sound received on three or more hydrophones in a common plane may be

used to locate the position of the source in that plane. These may be plotted as hyperbolic curves of equal arrival-time-difference measurements from two hydrophones. Additional pairs of hydrophones produce other hyperbolic curves that may cross to indicate the location of the source in that plane. See figure 2.

Plots of sound a· ·ival-time-differences from hydrophones in one plane provide information only in that plane, whatever its inclination. A bearing to the sound source may be indicated, but range information cannot be accurate unless the source is exactly in that plane. Only a slight departure from the plane of a two-dimensional array is needed to produce large errors in the indicated range to the source. To eliminate these ambiguities a three-dimensional position is necessary, even for sources in relatively shallow water.

Arrival-time-difference measurements of sound from hydrophones in an array in three dimensions can be used to locate a sound in three dimensions. Because of the complexities of hand plotting in three dimensions, this position is better calculated by computer. The difficulties of maintaining precision in attitude and dimension of such an array at sea require a method of concurrent calibration. Successive three-dimensional locations of a sound source in the water provides a track, and perhaps clues to behavior, as well as assisting in the identification of the sound producers.

A traditional rigid array of, say, 30 meters or more on a side seems unacceptably cumbersome unless fixed in position (bottom-mounted, for example). We experimented with a flexible, linear, three-hydrophone

array 300 m long suspended beneath sea ice in the Ross Sea (Schevill and

Watkins, in press), and were encouraged to suppose that under favorable

conditions a flexible 4-sensor (three-dimensional) array might yield a

shimmering picture that would be an acceptable improvement over our

usual ignorance. It developed that concurrent calibration of sensor positions

effectively nullified much of the shimmer.

As long as we are tied to sound arrival-time-differences we are

severely limited in range precision, for, as has been mentioned by Cummings

(1968, p. 6), the time-difference hyperbolae become increasingly asymp-

totic as range increases outside the array, but within a few diameters of the

array and with the refinement of a third dimension, the error is acceptable

against the uncontrolled estimates which one must make otherwise. Without

a second array (effectively permitting angular resection), it seems unlikely

that this can be improved.

Though there are obvious complexities, the following combination would

appear to provide reasonable location of relatively close underwater sound

sources: (1) a three-dimensional array of hydrophones to reduce range and

depth ambiguities, (2) a flexible non-rigid arrangement of the components of

such an array to permit its practical use at sea, (3) a system of concurrent

calibration to keep track of the varying dimensions of this array, (4) careful

measurement of the sound arrival-time-differences, and (5) computer cal-

culation of the locations of both the hydrophones and the sound sources.

The hydrophone array, the system of calibration, methods of arrival-time

difference measurement, and the associated calculation and computer programming

that we describe in the following pages have permitted some success in locating underwater sound sources.

Many accounts of acoustic location of biological sound sources by arrival-time-difference measurements on several hydrophones in an array have remained unpublished, and most of the published ones give only a. brief descriptions. Cummings (1968) gave a resumé of a few bioacoustic location experiments. The majority are two-dimensional locations either in plan or depth. Walker (1963) tracked finback 20-Hz sounds in the western N. Atlantic. Cummings, Brahy and Herrnkind (1964), Kronengold, Dann, Green, and Loewenstein (1964), and Cummings(1968) described various aspects of the acoustic system used off Bimini to locate and (with the help of a video system) identify a variety of underwater sound producers. Patterson and Hamilton (1964) outlined their efforts to track 20-Hz sound sources off Bermuda with separate horizontal and vertical (p. 141) arrays. Kibblewhite, Denham, and Barnes (1967) indicated acoustic locations of a variety of biological sound sources in New Zealand waters. In 1967 Wm. Whitney (MS.) used a vertical array in the Pacific to track sperm whale sounds. And our own previous experience includes tracking animals by their sounds with both horizontal arrays (one of which is briefly menticned on p. 149, Schevill, Watkins and Backus 1964) and vertical arrays (Schevill and Watkins, in press).

Three-dimensional locations have been attempted only occasionall,.

This is not only because of the complexities that are introduced by the third dimension into methods or mathematics used to derive the acoustic location, but also because sufficiently large hydrophone-separations in the third dir. .asion have been more trouble to arrange than those for the two-dimensional part of the array. Though some experiments originally might have been designed for three-dimensional location, later experience indicated that the data were useful only in two dimensions. Most three-dimensional locations generally have had relatively small hydrophone--separations in the third dimension, and therefore comparatively poor reliability of the source locations in that dimension. Anthony Perrone (personal comm.) tried to follow humpback whale sounds off Bermuda with an array having a third-dimension separation of about 60 m against the 550 and 1280 m two-dimensional plane. The array used by Winston Hole (1967) for tracking biological sounds in the eastern Caribbean Sea only had about 10 m of third-dimension separation against the two-dimensional spread of nearly 550 and 1130 m. The four-hydrophone array we generally use at sea is relatively small (30 m), but all hydrophone separations are about equal and thus accuracy of source locations should be the same in all dimensions.

Four-hydrophone array

The Array

The requirements of a sea-going three-dimensional array dictate its physical arrangement. At least four hydrophones are needed, and they must be separated (three-dimensionally) by about 30 meters; this is the shortest distance compatible with reasonable sound arrival-time-difference measure-

ments of biological sounds. To be useful, the array must be adaptable to the varying conditions experienced at sea, and it must permit rapid launching and retrieving so that moving sources can be followed. In addition, the array should be sufficiently flexible to permit its use from a variety of vessels.

The acoustical and electrical requirements are principally those that are common to any good portable listening system: broadband hydrophones, low impedance cables, quiet amplifiers, wide dynamic-range recording system, a high quality monitor, and a quiet power source. The components of the array and the ship itself must be silent in the water. Signals from each of the hydrophones are recorded simultaneously by in-line heads on four separate tracks.

The arrangement of the four-hydrophone array that has proved to be most convenient is as follows (figure 3): Two hydrophones are rigged fore and aft of the ship 30 m or more apart, a third hydrophone is hung from a float and allowed to drift 30 m or more away from the ship's side, and a fourth hydrophone is suspended 30 m or more below the side hydrophone from this same float. The approximate position of these beam hydrophones in the array is maintained by the ship's drift. To isolate the fore and aft hydrophones from the surge of the ship, it is best to suspend them from floats, and they may be held a few meters away from the vessel by poles rigged from bow and stern. The four hydrophones and their cables are prepared at a convenient time and made ready to be launched as needed. While underway, the cables with floats attached for the forward and after hydrophones are coiled and

stowed on bow and stern. The side and deep hydrophones also are attached to their float, and the cables are coiled and stowed amidships. The four cables lead to a listening position where the recording and monitoring gear is made ready. Thus, when needed, the hydrophones may be rapidly put overboard and recording started as soon as the ship is dead in the water.

The optimum depth of the array depends on local conditions, such as the water depth and location of a thermocline. Generally, the hydrophones must be suspended five meters or more below their floats to avoid surface noise. The deep hydrophone has to be weighted (wtih 5 to 10 kg) to ensure that it hangs below and is not pulled out horizontally by current or ship's drift. The active portion of a bathythermograph used in place of this weight permits an indication (to the depth of the array) of local water temperature structure and hence the sound velocity gradient with each use of the array.

Larger array dimensions(over 30 m) permit a better resolution of the underwater position of the sound source, but moving one or two hydrophones farther away without also increasing the separations between the other units does not particularly help the over-all resolution. Approximately equal distances between hydrophones give the best results in all directions.

·The location of the hydrophones relative to each other is calculated from arrival-time-differences of two sound pulses put into the water. Such an in-the-water calibration permits periodic reassessment of the hydrophone positions, and eliminates the need for the measurement of most of the array dimensions. Those measurements that are required may be made on deck: (1) the distance (depth) of the three surface hydrophones below their floats (the length of cable between hydrophones and floats), and (2) the distance

(the length of cable) between the deep and the side hydrophones. The weight on the deep hydrophone should be sufficient to keep this line approximately straight. As long as these hydrophone depths are maintained, no other measurements of the array are needed; all other dimensions may be calculated (more easily and more accurately).

The relative positions of the hydrophones in the array remain surprisingly constant over extended periods when the wind and current are steady.

Calibration

The calibration provides an acoustical means of locating the hydrophones by arrival-time-difference measurements of sound pulses in the water. The calibration procedure was developed together with procedures of time difference measurement, mathematical analysis, and computer programming for various array configurations. These all have undergone many changes. New procedures and calculations were developed to accommodate each variation of the array, resulting in a very flexible system that permits a wide variety of array shapes and relatively easy calibrations. Figure 13 shows a typical computer printout for such a calibration.

A sound pulse in the water from each of two sources (usually pingers) at unmeasured locations along the line between two of the hydrophones is used as the basis for locating the array hydrophones. Departures from this on-the-line position of the calibration pingers can be accommodated by the calibration program, but require other measurements. The easiest arrangement at sea places the calibration pingers between the bow and stern hydrophones, and makes this base line for the calculations approximately the same as the

ship's heading. Positions, then, may all be related to this base line and to one (usually the bow) hydrophone.

Arrival-time-difference measurements of the pulses from all four hydrophones are used to calculate the location (relative to the bow hydrophone) of the following in this order: the two calibration pingers, then the stern, the side, and the deep hydrophones. Positions are most conveniently based on an x-y-z coordinate system whose origin is at (the bow) hydrophone B. The line connecting (the stern) hydrophone A and hydrophone B coincides with the y-axis, and y increases in the direction of the ship's heading. The x-axis is perpendicular to the y-axis, and x increases to starboard. The z-axis is perpendicular to the x-y plane, and indicates depth (see figure 3A) as negative values of z.

Underwater sounds that are received on all four array hydrophones and whose characteristics permit arrival-time-difference measurements then may be located relative to hydrophone positions derived from a current calibration. The position of the sound source is indicated with reference to the same x-y-z coordinate system.

Sound arrival-time-differences

An ability to measure the differences in sound arrival time at the hydrophones is the key to successful location of calibration sound sources, array hydrophone positions, and also the sources of ambient sound. This is limited by : (1) the suitability of the sounds for such measurement, (2) the size of the array permitting large enough differences for reasonably refined location, (3) the resolution of the recorded sound, and (4) the measurement techniques.

(1) To be suitable for accurate arrival-time-difference measurement, underwater sounds must begin abruptly or have some sharply defined component, they must be relatively isolated in time or frequency from other sounds, and they must have sufficient intensity above background ambient to be measured. These are necessary not only to find the starting time for the sound arrival at one hydrophone, but also to compare arrival-times on different hydrophones.

Sharp front pulses from a nearby pinger represent the most easily measured sound. There is no uncertainty as to the start of these sounds, and they are not easily confused with ambient sound. The frequency spectrum of such pulses usually is distinctive enough to be differentiated from even louder animal sounds. Thus the calibration pulses usually pose little difficulty in arrival-time-difference measurement.

Animal sounds in the water are quite another matter. They seldom begin abruptly, appearing instead to increase in intensity progressively through-out the initial portion of the sounds. Thus, they often appear to rise out of the background ambient with no precise beginning. Many of these sounds are of varying frequency so that it may be difficult to isolate one from the others. A sound that has different frequency components from other sounds in the ambient can sometimes be isolated by selective acoustic filters.

(2) Ideally, a very large array should be used so that the arrival-time--differences can be relatively large for good resolution of the calculated position. Large size, however, involves long cables and more complicated methods of maintaining wide hydrophone separations. Practically, it is the length of the ship that dictates the size of a sea-going array because that is the hydrophone

separat'on that can be maintained easily. Long cables reduce the rapidity with which the array can be handled, an important asset in maintaining contact with animals at sea. The minimum size is dictated by the resolution that can be achieved in the recording process, and the ability to measure sufficiently large arrival-time-differences for adequate location. The smallest practical array for animal sounds seems to be one of about 30-m separations.

(3) The resolution of time differences that are possible during the recording process (usually limited to about 0.05 to 0.1 msec) is, of course, intimately dependent on techniques of measuring these differences. Often it is easier to measure accurately small time differences than to be precise over longer times. High frequency response, wow and flutter variations and relative alignment of signal tracks, repeatability in the whole record/playback process, all these limit the resolution of time. Usually, a means of averaging repeated signals is needed to assure usable accuracy.

(4) The method for measurement of sound arrival-time-differences that is outlined here has been the most convenient and the most accurate for a variety of sound types.

First, many sounds must be isolated from other interfering sounds by the use of selective filters. The filters inserted in the analysis system must be identical and adjusted to the same bandwidth and frequencies for all channels so that unequal delays are not inadvertently added.

Recorded sounds received on the four-hydrophone array are played back, and likely sequences are selected and rerecorded on a four-channel tape loop.

The four signals are best examined simultaneously on a four-trace oscilloscope with a variable, delayed sweep (such as the Tektronix model 535A). A sound on the tape loop may be used to trigger the start of the delay, which then may be adjusted to position the desired sound traces on the face of the oscilloscope. The signals may be displayed repeatedly (once for each revolution of the tape loop), for ease in comparing signals, and the traces may be manipulated, expanded and positioned at will. A judgment as to the utility of these sounds for location, the sequence of arrival, and relative time differences now may be made.

The signal that arrives first may be displayed now on one trace of a two-trace, two-sweep oscilloscope (such as a Tektronix 565). The sweep of this trace is triggered at the same time as the sweep of the four-trace oscilloscope, therefore the (first arrival) signal may be positioned (on both oscilloscopes) by the same delay control. The other three signals (2nd, 3rd, and 4th arrivals) may now be displayed sequentially on the other trace (of the 2-trace 'scope) whose sweep is controlled by a second delay circuit. This second delay permits the trace of these other signals to be lined up with (or superimposed on) the trace of the first-arrival signal.

The time difference between the start of the two traces of this oscilloscope then is the same as the arrival-time-difference between the two signals that have been lined up. An electronic counter (such as the Hewlett--Packard 5233L) may be used to indicate this time interval by beginning its time count with the start of the first sweep and stopping the count with the start of the second sweep.

Source location by computer

The measured arrival-time-differences are punched on cards and read by a program run on the XDS Sigma 7 computer at W.H.O.I. The computer provides a position in x-y-z coordinates. The calibration program produces a position for each of the hydrophones, and based on these positions, a location is calculated for each set of sound time-difference measurements. In addition, estimates are made of the errors that are likely in the computed coordinates, and relative intensity levels are calculated on the basis of relative distances.

The calculations were devised by Donna Ekstrand and are appended together with representative computer programming arranged by Mary Hunt. Additions and variations to the program continue to be made in order to increase its flexibility. The form shown here is the basic one in use as of February 1971.

Two dimensional locations

When three of the hydrophones in the array are placed in a plane parallel to the surface, the two-dimensional information can provide an approximate source location in plan. With a shallow array and shallow sound-sources these positions may nearly coincide with the real three-dimensional location. In fact, since deeper hydrophones often are plagued by sound distorting thermoclines, many sounds are not received well on the deep hydrophones, and only the "surface data" then is available for calculating locations.

The difference in location provided by two- and three-dimensional information varies with distance and depth of the source relative to the array. The two-dimensional position ignores depth differences and assumes that the sound is in that same plane. This position lies on the same bearing as the

three-dimensional location, but may be at quite a different distance from the array. Unless they coincide, the two-dimensional position is always closer than the three-dimensional one.

The inability to receive satisfactory sounds on deep hydrophones is a common enough complaint so that usually we arrange the array to keep one plane parallel to the surface. Thus, (except for sounds that arrive first on the deep hydrophone) "surface data" always is available, and the computer program has been made to give both a two-dimensional location and the three-dimensional one (see sample printout, figure 14).

Errors affect range more than bearing

The method of deriving the distance of a sound source by arrival-time--difference measurements has inherent uncertainties particularly for more distant sounds. This can be demonstrated most easily when plots of arrival--time differences are made of remote sounds. Such plots (figure 2) form hyperbolic curves whose outer segments straighten with distance and become almost parallel (asymptotic) to other curves for the same sound (from other hydrophones); the curves intersect at small angles only. Because of these small angles, a variation in any one time-difference measurement changes the distance to the point of intersection by an amount that increases rapidly with distance from the receivers. This may be readily noted in two-dimensional plots; in three-dimensional work the effects are compounded. Curves that cross at large enough angles to fix a location with precision are those for relatively close sound sources only. Small varaitions in the measurement of sound arrival-time-differences, therefore, can be responsible for relatively large differences in

indicated range (particularly for distant sounds). This is true not only for plotted sound arrival-time positions, but also applies to calculated locations; it is the result of the sound arrival-time relationships.

The same relationships that militate against precision in range, however, make for much better readings of bearing. A variation in any one time-difference measurement changes the azimuth indication by only a small amount. Thus the direction of a sound source may be changed but little by relatively large errors. The result of error usually is uncertainty in range, while the bearing may be largely unaffected (see figure 4).

In three-dimensional work these relationships continue for each plane, modified of course by information from the other hydrophone planes. The potential range uncertainty and good direction are true in all planes. Thus, the indication of depth for a near-surface source is more reliable than its distance from the array, since (within a few diameters of the array) depth is indicated here as a function of azimuth in a vertical plane (see figure 4, and discussion of accuracy in the next section).

Accuracy

The reliability of positions indicated by the array is difficult to assess. The entire array is nonrigid, components are joined by flexible cables, and there are no fixed points of reference. Yet sounds that provide good time-difference measurements seem to give reasonable positions with little wild scatter (rarely locations in the air or below the bottom). The range uncertainty (as noted above) increases rapidly with distance, though the apparent reliability of directional information is impressive.

Theoretically, if the array had not moved since its last calibration, and the sound were close-by as well as ideally suited (for arrival-time -measurements), then source location could be as accurate as the time-difference measurement permitted. In our experiments thus far, the practical resolution of arrival-time-differences has been 0. 05 to 0. 1 msec. This would make a possible error in each measurement for sound sources within the array of up to 15 cm (potentially cumulative for all the measurements required for each location) We estimate that our best positions for close sound sources are within 1 m (as for sound number 153 in figure 4).

The effect of error on the calculated positions for sound sources has been studied in detail in order to assess the utility of the array in different configurations. Tests also were made of the effect of error in locating the hydrophones in the array and then of the cumulative effects of these calibration errors on source position. All such errors produce immediate differences in apparent distance (to sources outside the array), but very little change in direction.

For a source within the diameter of the array, displacements of computed position which are due to errors of time-difference measurement vary with the location of the sound source relative to the individual hydrophones. Within a 30-m array, 0. 1 msec (about 0. 5% of maximum arrival-time--difference measurement) of error in all time differences produces cumulative errors of up to one meter in the location of a source. Proximity to any one hydrophone decreases the displacement from error in time-measurement on sounds from that unit and increases the displacement vector produced by errors

on the more distant hydrophones. Thus, the potential shift in position (within the array) due to equal error in all time measurements will be greatest in a direction opposite the farthest hydrophones (see figure 5).

For a source ou. ide the array, errors in arrival-time-difference measurements produce displacements of position that lie along a line oriented toward and away from the array. When errors are made on the signal from one hydrophone only, this line points toward that portion of the array opposite this hydrophone. The larger the error the greater the displacement along this line (see figure 5). Thus, a series of positions (for a sound) which lies along a line pointing toward one side of the array may be indicative of error in the time measurements of sound received by the hydrophone on the opposite side of the array (from the error line).

Errors on more than one hydrophone produce a displacement in position (of a sound outside the array) along a line directed toward (or away from) a point between the hydrophones; this point is located by the relative magnitude of the errors. Errors in all measurements for one sound combine to produce displacements in a direction generally toward (or away from) the center of the array. At any distance from the array of course, apparent angular resolution decreases so that all errors appear to create displacements in position which form a fairly straight line, oriented generally toward (or away from) the array. Therefore. though the range to a source may be ambiguous because of timing errors, the line formed by these erroneous positions reliably indicates the direction to the sound source.

A constant increment of error on the same hydrophone produces a larger displacement of position with distance, and the same errors on different hydrophones produce varying amounts of displacement depending on both relative direction and distance to the source. The results of timing errors of the same magnitude on different hydrophones is shown in figure 4.

Timing measurement errors in the calibration may operate cumulatively with errors in arrival-time-differences of a sound to form much larger displacements of position. Again these cumulative error positions lie on a line in the general direction of the true position of the sound source relative to the array (figure 4).

Time errors that result from bias of an individual or a piece of equipment which tends to make all measurements slightly positive or negative, however, generally seem to cancel out. This is true also of most random small errors. Sometimes, however, random errors may act cumulatively and produce relatively large differences in the computed position, especially true of a distant source (see below).

The same magnitude of error produces varying amounts of position displacement for sources located at different distances from the array. See figure 4. The extremes in these plots are the result of (a) the largest errors (in arrival-time measurement) likely with good sounds, (b) such errors in all time measurements both of the calibration and of the sound, and (c) the arrangement of these errors to produce the greatest displacement possible. All positions in this figure marked with the same numbers have equivalent errors of ±0.1 msec in all time measurements. The differences in the effect of these cumulative "worst-case" errors relative to the distance of the source

from the array may be noted: Within the diameter (40 to 45 m) of the 30-m array, the maximum spread due to error is less than one meter; within a second diameter's distance, the maximum spread (in range) is nearly 10 m; and within a third diameter's distance, the maximum spread is about 25 m — all with the same magnitude of measurement error. The displacement of position due to the same amount of error increases rapidly with distance outside the array.

These relationships are true of all the hydrophone planes. The plots shown in the figures have been of the surface (x-y) plane, and since it often has been convenient to arrange three of the hydrophones in this plane, these plots are valid representations of the usual surface view. Three other hydrophone planes also are present in the geometry of the array, and the relationships within those planes work the same way as that described for the surface. The location coordinate system (x, y and z in perpendicular planes) is independent of (though it may coincide with) the hydrophones planes, but the calculated source locations are the result of the interaction of all these planes.

Depth indication of a sound source may be more accurate than its distance from the array because the depth (of relatively close sound sources) represents a bearing in the vertical plane. As long as a source is outside the array and within about the same depth as the array, the effect of range error on depth will be relatively small and the indicated depth will be the result mostly of the vertical angle to the source.

Additionally, the computed positions include an estimate of the potential reliability of the calculated coordinates for sound sources. In the location program, an error of a specific amount is inserted purposely in each time difference measurement and the potential variation in each coordinate is calcu-

lated. An error of 1 msec is commonly used in this error program and represents 10 times the usual uncertainty in measurement of good signals. The variations in each coordinate that is due to this magnitude of error provides an assessment of the relative reliability of each coordinate in the location. This may be calculated and given in the computer read-out for source locations (see figure 14).

Intensity measurements

Calculations of intensity level also are included in the computer program. Since distances to a sound source from each of the hydrophones may be calculated from their positions, it is possible to compute the differences in signal level that should be received by the different hydrophones. We assume spherical spreading and isovelocity water and compare all intensity levels to the level on the hydrophone at which the sound arrived first. The level at this first arrival is designated as O dB. The intensity of the sound source then may be indicated, also relative to the first arrival. Basing the calculations only on distances relative to the first arrival hydrophone permits the individual variables (attenuation and amplification in the system, hydrophone receiving sensitivity, and the received intensity level) to be included as needed in assessing source levels. The flexibility of the computer program thus is retained, permitting its use with a wide variety of system components.

The intensity calculations give the levels that should be received by the different hydrophones if the sound source were at the computed position. Comparison of these calculated differences with the differences actually received by the hydrophones may give another assessment of the reliability

of the computed position. Except for close sounds, distance (from the array) is the least reliable parameter in computed source locations, and since the calculation of source level is based on distance a similar scatter may be expected in the intensity levels derived from the array. The potential variation in calculated level, however, may be too great to be useful (see figure 4), since level differences due to changes in position are not linear but vary with the square of the distance.

Other array configurations

A variety of array configurations and ways to maintain the needed hydrophone separations have been examined. The array described above (figure 3A) is the product of experience gained through trials with other arrangements. Some of these had particular advantages for special purposes and they are listed here to illustrate the flexibility of the array, calibration and computer combination:

Fixed arrays with known hydrophone positions. No configuration calibration is necessary and only the location portion of the computer program need be used to locate or track sound sources.

Fixed arrays with unknown hydrophone positions — such as an anchored array or one composed of hydrophones dropped to the bottom in shoal water with the hydrophone for the third dimension buoyed near the surface. Calibration allows these units to be located relative to one hydrophone and then sound sources may be located relative to the array.

Large array dimensions (up to 2 km) have been tried, but though the sound-arrival-time differences are advantageously large, only very loud sounds

can be detected on all hydrophones. In addition, large time differences have proved to be more difficult to measure with precision (using the usual laboratory equipment) than the shorter time spans, consequently relative accuracy does not always increase with the use of larger arrays.

A right-angle array(fig. 3B) permits more rapid assessment of source position. Three hydrophones form each of the three planes which are at right angles with each other; the center of the coordinates is at a common hydrophone (B). The right-angle configuration simplifies the interrelationships between planes, and permits a rough estimate of source location by the use of plots of arrival-time-difference hyperbolic curves for each plane. However, the right angles are difficult to maintain at sea.

Arrays have been formed with other means than the length of the ship to maintain the separations between hydrophones. These have included various methods of actively pulling on the cables with rowed skiffs, dinghies with slow-turning motors, wind and water vanes, and self-propelled robots. All of the methods of propelling a towing vehicle produce some sound and complicate the array. During conditions of little wind, however, some such method of separating the hydrophones for longer times is useful. With periodic calibrations, the hydrophones can be pulled into position at intervals and then quietly allowed to drift toward each other during listening periods.

Rigid structures in the water to hold the hydrophones apart are feasible if set-up time is not critical. Floating pipe-structures up to 60 meters long have been used (figure 3C), but all such arrangements take so long to set that anchored arrays (where feasible) probably are easier.

__Boom extensions to make a small ship longer__ are useful as long
as they can be kept out of the water (to avoid transmitting deck noises and
being damaged by the sea). Even in air long booms must be strong to with-
stand the lateral pull of the hull from wind, current, and the roll of the
vessel.

Calibration variations

A variety of calibration arrangements will work with the computer
program; we have described only one. The two calibration signals may be any
sharp-front sounds (as from a pinger or from a fire extinguisher partly immer-
sed and struck with a hammer) since it is arrival-times that are measured to
calculate the positions.

The calibration signals (figure 3) that are most easily utilized are
those from sound sources located between two of the hydrophones (which are
then designated A&B). In fact, the easiest arrangement of all is for the
pingers to be placed with these hydrophones, or alternatively, to drive the
hydrophones themselves to produce pulses for calibration. Most hydrophones,
however, are not capable of transmitting very loud sounds. The farther apart
the pingers are along the AB line, the more accurate will be the location of the
outboard hydrophones (C&D).

The calibration pingers may be located off the AB line but they must
not be farther apart than the hydrophones. Their distance from the line between
the hydrophones must be known, but their separation from each other need not
be. This separation is calculated by the computer. Mostly we have used three
positions for the calibration pingers: (1) On the AB line, (2) a meter or so

above and fastened to the cables of two of the hydrophones, and (3) a variable
(though measured) distance from the array and at a known depth. When using
a floating boom as separator for the array, usually it has been more convenient
to keep the pingers alongside the ship, even when the array was allowed to drift
30 or more meters away (to reduce the sounds of hull water noise). Here the
tethering lines to the array were measured to provide the distance from calib-
ration pingers to the line between hydrophones A&B (figure 3).

Array results

Pingers. Early experiments with the array were confined to track-
ing pingers or other sound sources towed or lowered near by. In such tests,
the importance of accurate calibrations was demonstrated. When something
was wrong with a calibration the source positions were poor, but when the
calibrations were good the source tracks through the array were reasonable.
Attempts to measure by line the distances to a source within the array consis-
tently gave answers that were within a meter of the calculated positions. It was
interesting then to find that this meter variation matched the spread that could
be expected with the usual precision in time measurement (see section on
accuracy).

Passing ships. Occasionally ships or small boats would come by
while the experimental array was set and attempts would be made later to
reconstruct their tracks from the acoustic data. This was successful only when
the same discrete sounds from engine or bow wave could be found on all hydro-
phones and the individual sounds started suddenly enough to measure arrival-
-time differences. When sufficiently discrete and sharp-front sounds for good

location were lacking, however, it was possible sometimes to obtain a bearing
from the sound even though the indicated range obviously was wrong. These
bearings correlated well with compass bearings and times taken during the ship's
passage.

Echosounder. On one occasion, a Coast Guard cutter passed at
about 600 or 700 m. The pulses from her echosounder, though nearly masked
by noise, were used to draw a short track for the ship. Bearings to positions
on the calculated track matched those taken at the time, and the distances to
the ship varied only by about ten percent which was remarkable for sounds at
this distance. But the ship appeared to be running along the bottom in 40 m of
water. We interpret this as the result of the signal coming by way of a bottom
reflection instead of from the ship's narrow beam transducer.

Deck noises. Sounds made aboard the ship used to support the array
all plot to likely positions. Sounds made at the same location on the ship plot
very close together near the surface, and sounds made at different locations
on the vessel plot at similarly separated positions. One series of sounds made
by equipment dragged from one side to the other across the deck can be
followed in a somewhat similar underwater track, though we do not know the
paths that the sounds took to reach the water.

Bottom sounds. An occasional snapping sound in the underwater
ambient can be plotted to give a position on or near the bottom. We assume these
are crustacean sounds.

Eubalaena tailslaps. Tailslaps on the surface of the water by the
flukes of right whales (Eubalaena glacialis) consistently plot at the surface even
when the animals are 300 m or more distant. Direction to these sounds shows

good correlation with the direction to the tail-slapping whale that was observed from the ship. The distances also seem reasonable especially for the closer animals (figure 6). The tailslap is a good sound for time-difference measurement so these distances may be correct.

Eubalaena moans. The low frequency moans produced by the right whale underwater are not good for location by the array. They generally do not begin abruptly; instead they increase in intensity during the first part of the sound so that they usually appear to rise out of the background ambient. In addition they often sweep in frequency, reducing the usefulness of band-pass filters for isolating a portion of the sound. The whales produce sounds only occasionally, so that there is little opportunity to compare positions of adjacent sounds. Few sounds were heard when animals were near the ship. Both right whales and porpoises (Lagenorhynchus albirostris) occasionally passed within a few meters of a hydrophone and sometimes through the array. But these passages usually were silent ones.

During a cruise on R/V Gosnold in Cape Cod Bay (May 1970) about 20 of these whales (Eubalaena glacialis) were recorded over a 3-hour period with the 30-m three-dimensional array. Some sound positions were plotted successfully, and indicated a preferred depth of 15 to 20 meters (figure 6). At the time, of course, we had no idea as to the location of any of the whales producing sounds, so no confirmation of the acoustic positions are possible. Yet sometimes a surfacing whale (figure 1) appeared on a general bearing that was indicated later by the sound position. We do not trust the distances in most of the acoustic positions for the right whale sounds; undoubtedly, they vary widely due to the impossibility of precise time measurements on these

sounds. But both bearing and depth probably are good.

L. albirostris squeals. Lagenorhynchus albirostris, a porpoise
that visits Cape Cod waters in the spring, produces squeals that are between
8 and 12 kHz. Some of these sounds also were used in acoustic location of
the animals (figure 7). Usually, their squeals do not lend themselves easily
to good time-difference measurement, but occasionally a step in frequency
or a sudden change in sound amplitude could be used to locate these porpoises
(figure 12). Recordings made during the 1970 Gosnold cruise produced several
sequences in which nearby animals could be acoustically located. One series
that seemed like an ideal one at the time of recording, was of several porpoises
apparently annoying a right whale — swimming around its head while the whale
accelerated rapidly at the surface. This activity occurred during a chorus
of loud porpoise squeals and repeated short whale moans. Disappointingly, the
porpoise chorus had so much overlapping sound that direction was all that could
be derived from either the porpoise or whale sounds, and then not sufficiently
simultaneously to place both on the same bearing.

Balaenoptera 20-Hz sounds. 20-Hz sounds were recorded from a
small group of finback whales (Balaenoptera physalus) also during the 1970
Gosnold cruise. An indication of direction to the whales' position was obtainable
even though the computed locations were obviously wrong (figure 8). Since
the wave length of these sounds was about two-and-a-half times the hydrophone
spacing on our array, we did not anticipate being able to measure their
arrival-time-differences. Sounds from ship traffic and the gradually increas-
ing intensity of the beginnings of these sounds further complicated the measure-
ment. Good correlation in the direction to the animals was obtained, however,

by aligning the slow sweep in frequency (22 to 19 Hz) within these one-second pulses. The depths indicated for these sounds were 19 to 24 meters, and probably are a result of the inability to measure finer differences (than approximetely equal arrival-times) on C and D hydrophones. Distances to the calculated positions are much too short since the whales were observed to be 1 km or more distant and slowly moving away. Several positions were calculated from different parts of each signal; those that plotted differently, as in sound number 614 and 625 (figure 8), show progressions away from the array on a line oriented roughly from the center of the array. This would indicate perhaps that the progressions reflect the real motion of the source away from the array. Except for sound number 929, the general progression of successive sound positions also is away from the array. The orientation of the line of sound positions is away from that portion of the array opposite hydrophone A. and perhaps may indicate errors in time measurements on that hydrophone, as in figure 5.

Many of the results from these experiments remain puzzling and are not easily explained by the long wave lengths relative to array size, the local environment, the array configuration, or instrumental foibles. Sound intensities often were very different on successive hydrophones (figure 9 and figure 11), dramatically reducing confidence in the results of source-level calculations. Phase reversals of the signal sometimes occurred on one or more hydrophones independently of the others. This was marked especially in the low frequency pulses from finback whales (figure 9).

Disadvantages of the array

Analysis time. The length of time required for analysis is the main
difficulty in successful use of the array and its accompanying system. Even
with pinger pulses which can be nearly identical, the measurement of sound
arrival-time-differences is tedious and time consuming. Each additional
type of sound must be treated differently and human judgment seems to be
necessary. Just to pick the same sound out of the background ambient on
successive hydrophones and then to find the same spot on each of these sounds
for measurement, may be a difficult task (as in figure 9 and figure 12). Time
is required to make these judgments and so recorded data (rather than live)
is necessary to permit repetition and trial of different analysis methods. A
spectrographic portrayal of the sound is often useful to determine the frequency-
-band needed to isolate the sound from the rest of the ambient.   Auto-correlation
and signal recognition techniques by computer would be useful especially if a
large number of nearly identical sounds were being tracked. But the wide
variety of sounds in the usual biological ambient obviates much of the advantage
in these techniques.

Intensity variations between the different hydrophones of an array,
further complicate its use. In shallow water, especially, this is a nuisance.
The combinations of surface and bottom reflections with attendant phase-rever-
sals and constructive or destructive enforcement are complicated by sound-path
variations. At its worst, these effects can be added to those of a thermocline
which may effectively isolate deep hydrophones from near-surface ones. The
same 300 -Hz right whale moan from Cape Cod Bay is shown in all four parts
of figure 9. Each was received by a different though essentially identical

hydrophone channel in the array. Amplitudes of these traces have been adjusted to more or less equalize the visual presentation. The hydrophones were separated by about 30 meters (as in figure 6). By intensity alone, it was impossible to find the same spot on all traces of the sound, yet aurally the sound was quite recognizable on all channels. Listening systems with only one hydrophone give an impression of the intensity of underwater sounds that are valid only for that location; the intensities especially at lower frequencies may be quite different a few meters away.

Equipment failure becomes a hazard as listening systems are complicated by multiple channels. The failure rate seems to increase by some factor that is larger than the linear increase in components. Presumably, this results at least partly from the necessity for the operator to share his attention with all the channels. Failures to produce the best possible results are most serious during field recording, but may be noticed also in playback and analysis (which, however, can be repeated). Though we have tried to keep the entire 3-dimensional system as simple as possible, the likelihood of failure to obtain the desired results increases drastically with each added complication. We have been successful in producing consistently good field recordings on single hydrophone systems, but find it frustratingly difficult to maintain the same excellence in four-channel recording.

Conclusions

The non-rigid, floating, four-hydrophone array can give accurate three-dimensional locations for certain underwater sound sources, particularly for those that are close. For more distant sounds, the array can indicate good

directions (bearings) but range is liable to be uncertain. For sound sources that are within the depth range of the components of the array, the depth information also should be good.

The array and its computer analysis are made more flexible by the system of concurrent calibration which permits periodic reassessment of hydrophone positions.

Acknowledgements

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Figure 1   Right whale diving near array.

A right whale (Eubalaena glacialis) dives near the outboard

buoy which supported both side (C) and deep (D) hydrophones.

Generally these close approaches were silent.

25

Figure 2   Arrival-time-difference hyperbolic curves.

The hyperbolic curves are plotted in the plane of hydrophones A, B, and C and represent the arrival-time-difference measurements on pairs of hydrophones for sound number 245 and sound number 121. The curves cross at the locations of the sounds and form smaller angles with increasing distance from the array.

Arrival–time–difference
Hyperbolic Curves

Distance in meters

Time in msec
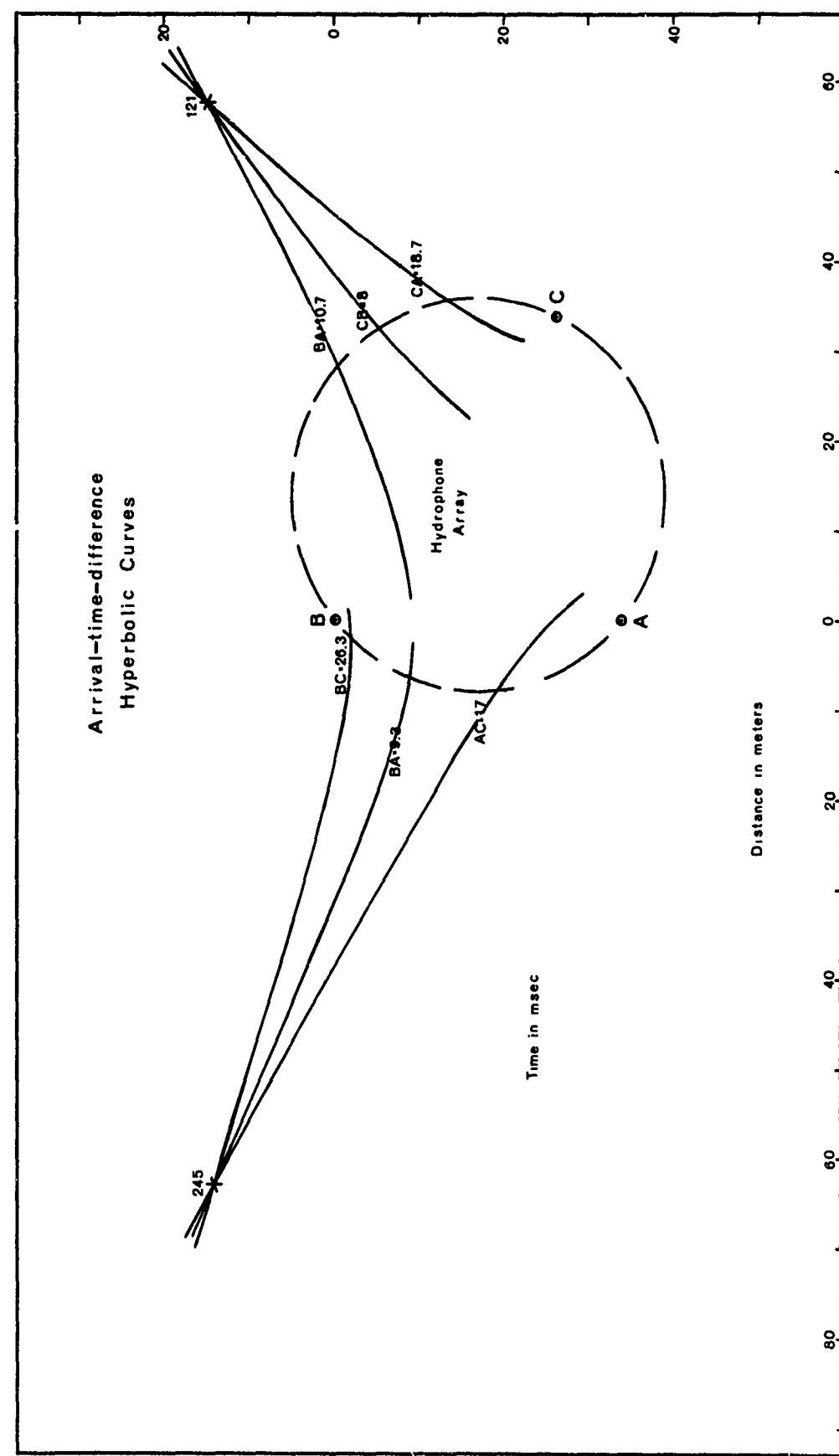
Figure 3   Arrangements of the array.

(A) The usual sea-going arrangement, (B) a ninety-degree configuration and, (C) a floating-boom arrangement used with smaller boats. In each case the wind is from left to right, allowing the vessel to drift away from the buoyed hydrophones.

90° Array Configuration
3B

3C **Floating Boom Array**

3A

**4-Hydrophone Array**

Sea-going Array
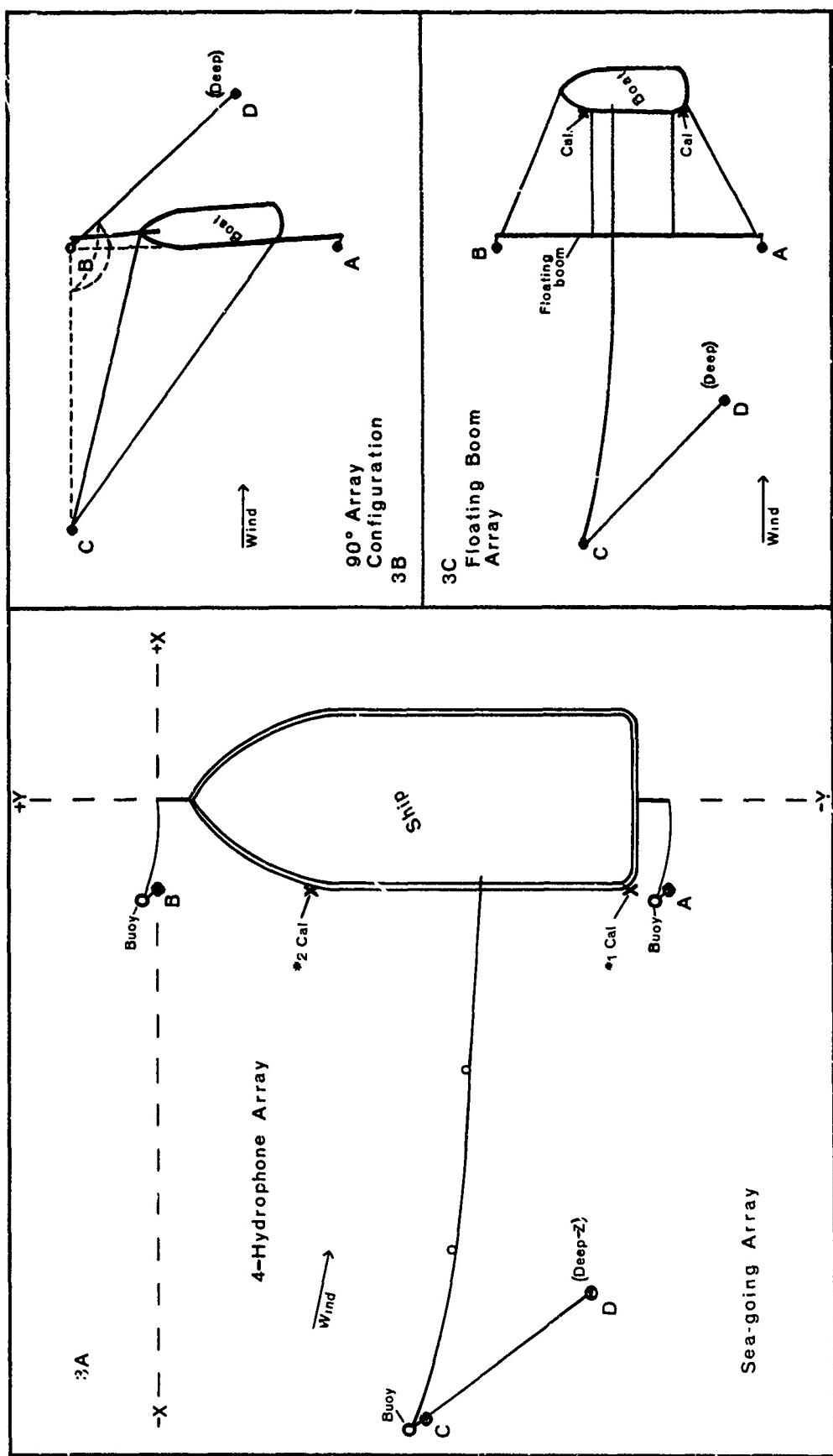
Figure 4   Cumulative errors in both calibration and data.

Positions on both sides of sound number 245, 153, and 121 are

the result of error in all time measurements of the calibration and

data sequences for those positions. The extremes represent the

worst possible combination of errors of this magnitude. All the

errors in time measurement are either plus or minus 0.1 msec,

and are, of course, cumulative in their effect. Positions marked

with the same number have equivalent cumulative errors; thus

postion 4 of sound number 245 has the same amoun of error as

position 4 of sound 153 and position 4 of 121. Note the increasing

effect of error with distance from the array and the orientation of

these displacements in position. Depths are indicated by parentheses,

and at these ranges are affected little by error. The potential

variation of intensity from the extremes in these positions is calcu-

lated to be 3.2 dB for sound number 245 and 1.7 dB for sound

number 121.

CUMULATIVE ERRORS IN BOTH CALIBRATION AND DATA

Hydrophone Array

153

(15.1)

121

(34.8) D

C

B

A

Ship

1
x (6.5)

121 x 2 (7.0)

x 3 (7.3)

4 x (7.3)

1 x (8.4)

2 x

245 x (9.7)

3 x

4 x (10.6)

121
Intensity difference between 1 and 4 = 1.7 dB

245
Intensity difference between 1 and 4 = 3.2 dB

Positions with the same numbers have equivalent cumulative errors

All errors = 0.1 msec in each time measurement

Parentheses indicate depth

Distances and depths in meters
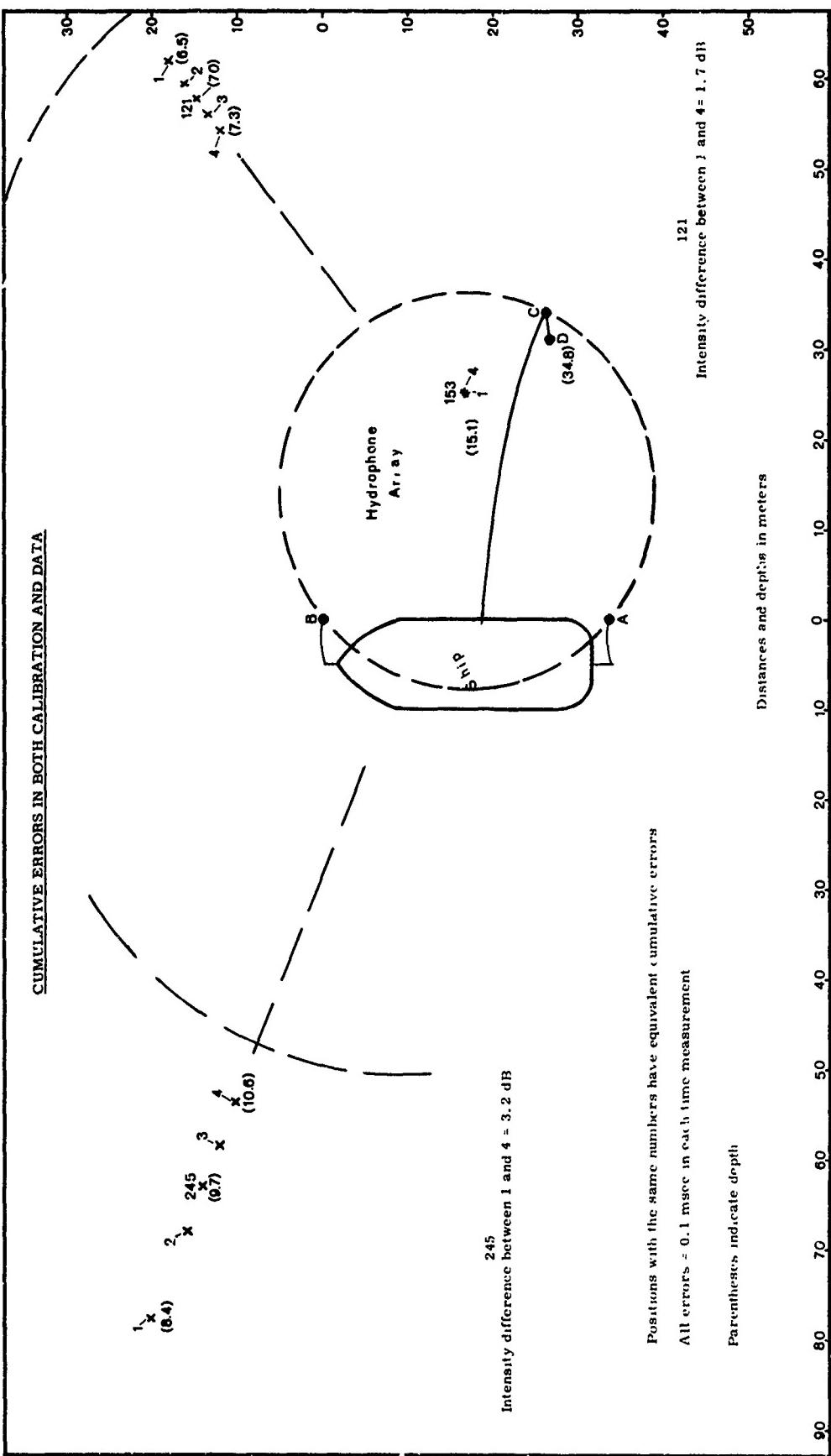
Figure 5   Data errors on one hydrophone.

Position on both sides of sound number 245 and 155 are the result of error in time measurements on only one hydrophone at a time. The errors are in increments of 0.2 msec, as indicated by the table, and are labeled according to the hydrophone which had the error ($C1$, $C2$, $C3$ = errors on C hydrophone). Positions marked with the same number have the same magnitude of error, thus $c1$ and $a1$ both have 4 msec of error. Note the increasing effect of error with distance from the array, and the orientation of the error positions. Errors on any one hydrophone produce displacements in position that fall on a line oriented toward (or

away from) a part of the hydrophone array that is opposite the hydrophone.

DATA ERRORS ON ONE HYDROPHONE

Hydrophone Array

Ship

Positions with the same numbers have the same magnitude of error.

Errors are in increments of 0.2 msec on only one hydrophone at a time:

c1= +4 msec	a1= ±4 msec	b1 = -4 msec
c2= +2 msec		
c3= -2 msec		
c4= -4 msec	a4= ±4 msec	b4 = +4 msec

Hydrophones responsible for errors indicated by lower case letters.

Parentheses indicate depth.

Distances and depths in meters

Figure 6   Eubalaena glacialis, acoustic locations

Plotted positions obtained for the sounds from very close right whales, Eubalaena glacialis, show their apparent preference for depths of 17 to 20 m. Tail slaps on the surface of the water provided good sounds for acoustic location and one very close blow was successfully located. The positions connected by broken lines are related in time and may be from the same whale or group of whales.

Eubalaena glacialis

Distances in meters
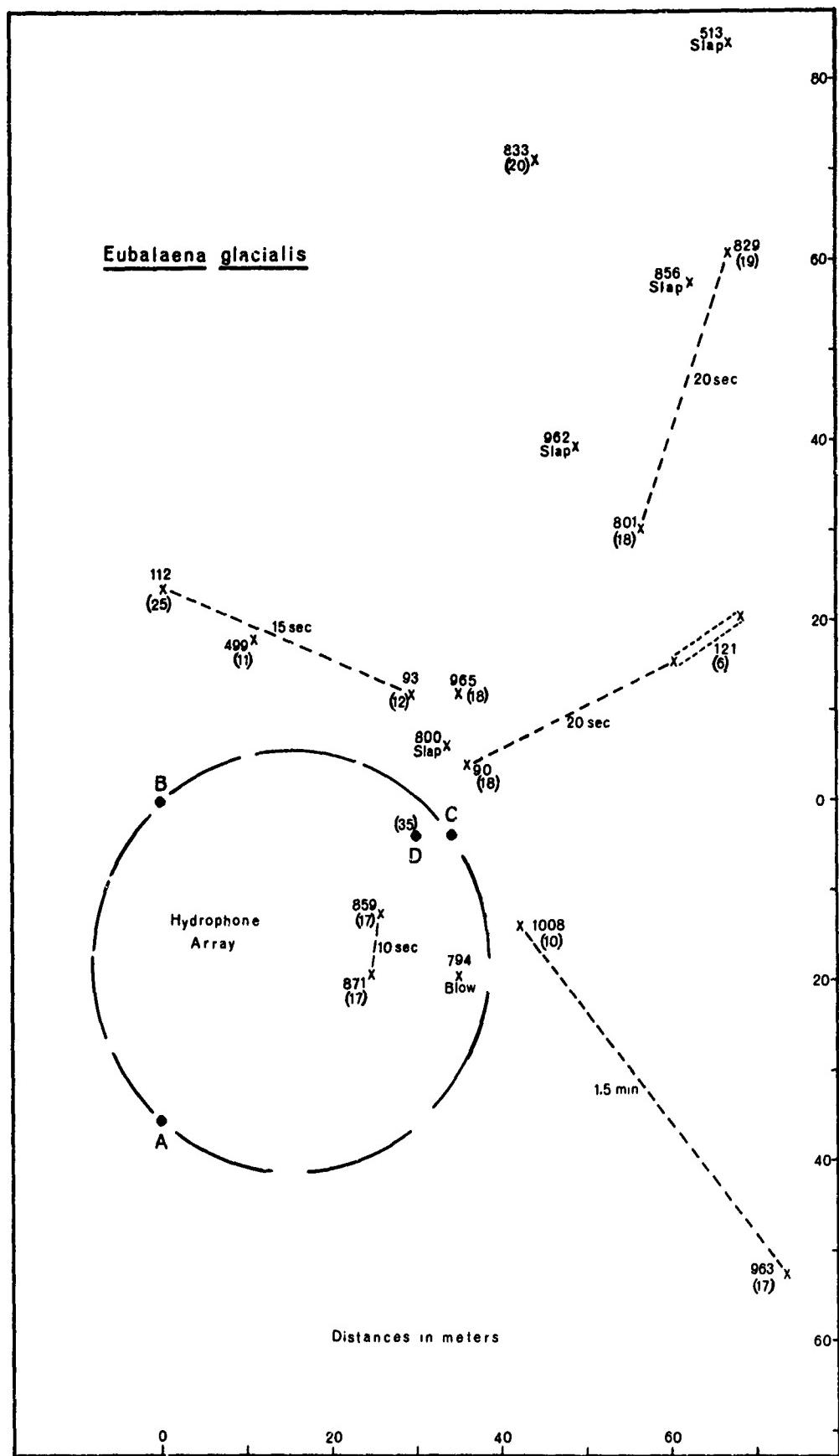
Figure 7   <u>Lagenorhynchus</u> <u>albirostris,</u> acoustic locations

Some of the squeals from rearby porpoises (<u>Lagenorhynchus</u>

<u>albirostris</u>) were successfully located.   Plotted positions connected

by broken lines are sequential and may be sounds from the same

animal or group of animals.

Lagenorhynchus albirostris

166
x
(22)

237
x
(11)

870 x (13)

165
x
(19)

155
x
(30)

10sec

275
x

153
x
(15)

Hydrophone
Array

2sec

168 x
(19)

1.5min

236
x
(20)

167 x
(4)

C ● (5)

(35) ●
D

B ● (5)

(5) ●
A

30 20 10 0 10 20 30 40 50

0 10 20 30 40 50 60 70 80 90 100

Distances in meters

Parentheses indicate depth
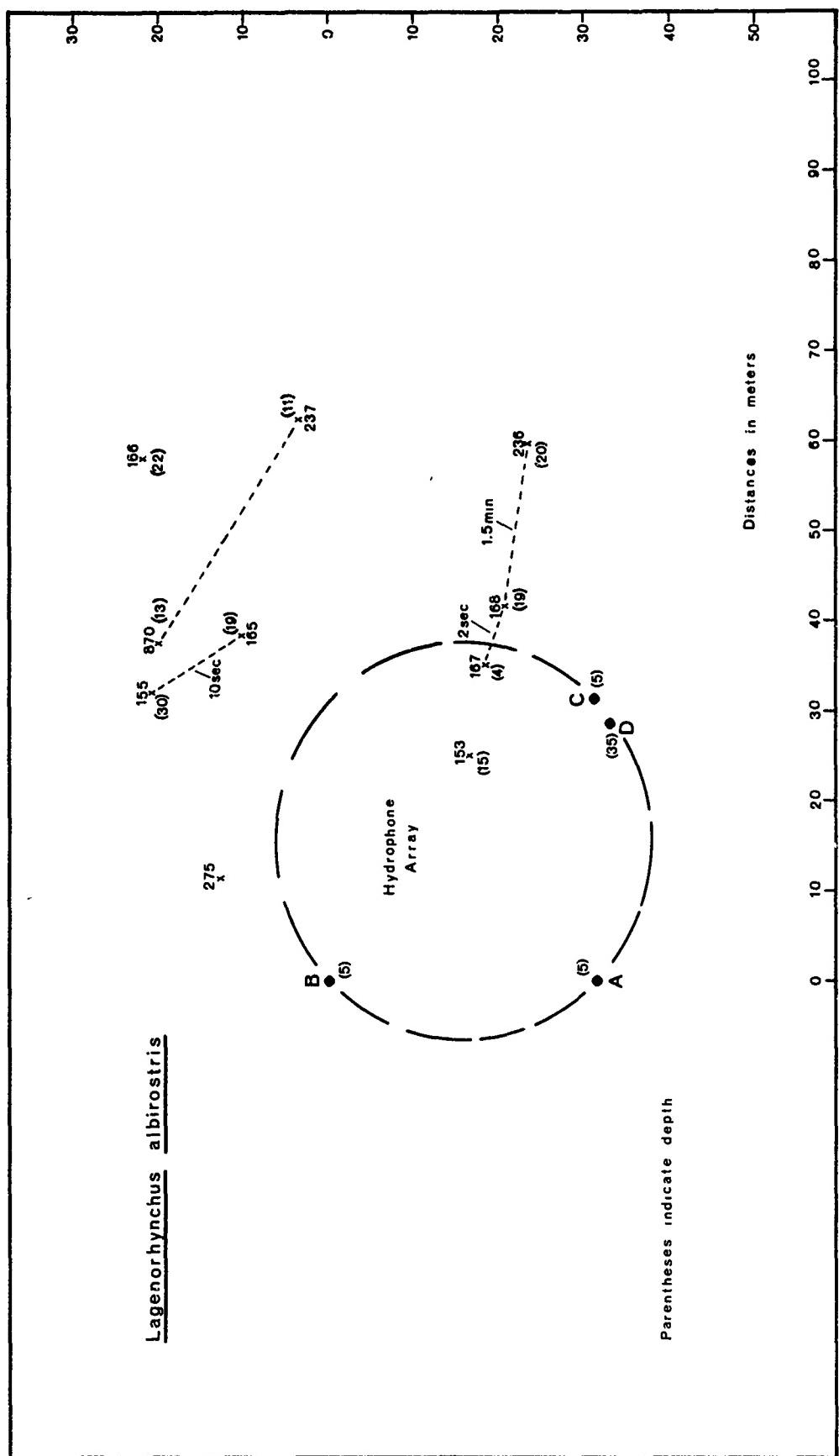
Figure 8   <u>Balaenoptera physalus,</u> acoustic locations

Positions for 20 Hz sounds from distant (1 km or more)

finback whales, <u>Balaenoptera physalus,</u> plot much too close to

the array. But, though range is erroneously indicated, direction

is approximately correct. The arrival-time-difference measure-

ments used for these plots were obtained by alignment of the slight

frequency sweep (22 to 19 Hz) contained in the sounds.

Balaenoptera physalus

Distance in meters
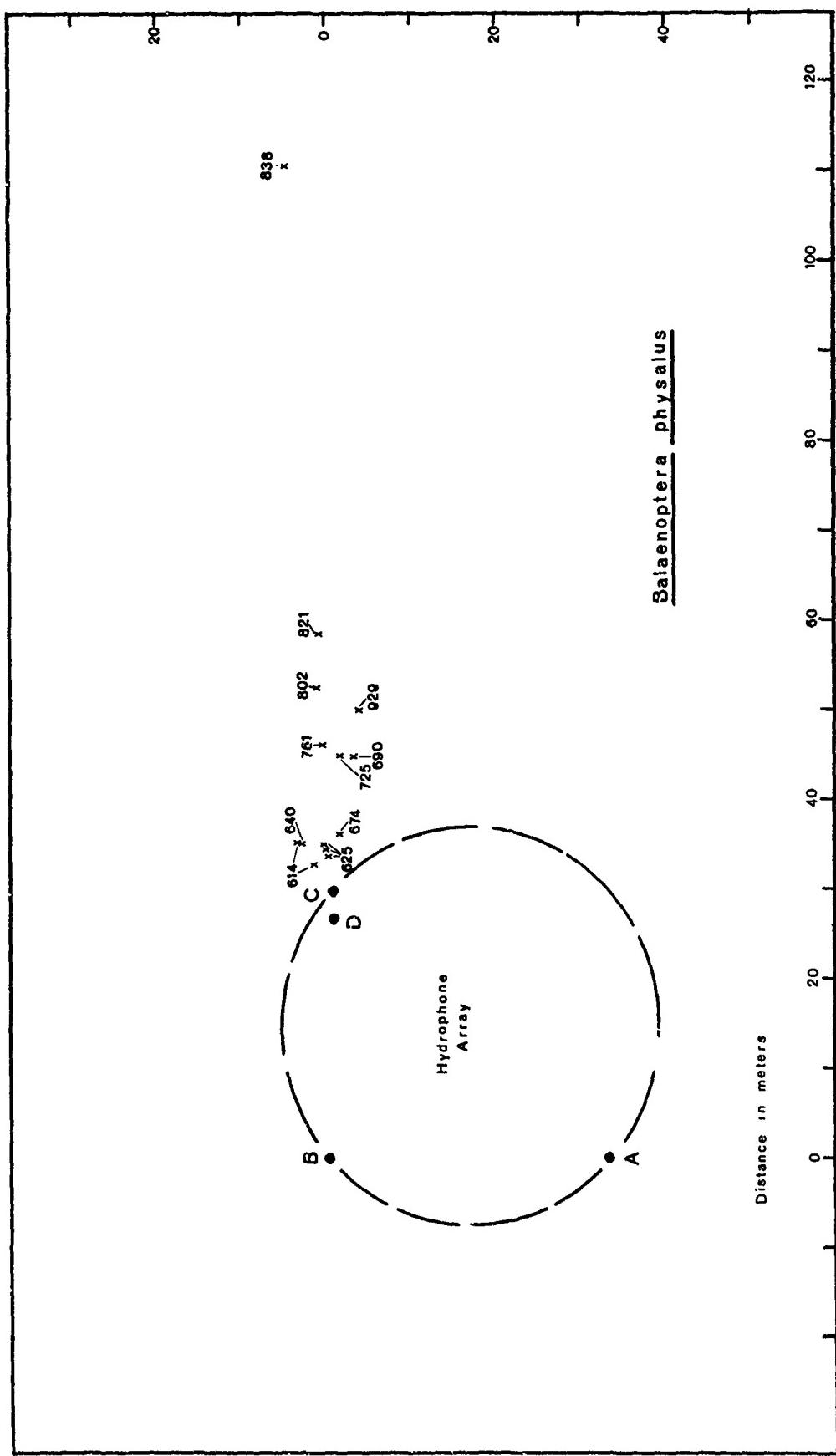
Figure 9   Oscilloscope photograph of sound on four channels

The four traces illustrate the extreme variability of sound intensity in shallow (50 m) water. This is the same 180 Hz right whale moan recorded from the four array hydrophones (ABC&D), approximately 30 m apart. Scale = 0.1 sec/division. Note how difficult it is to find the same part of the sound in all traces.
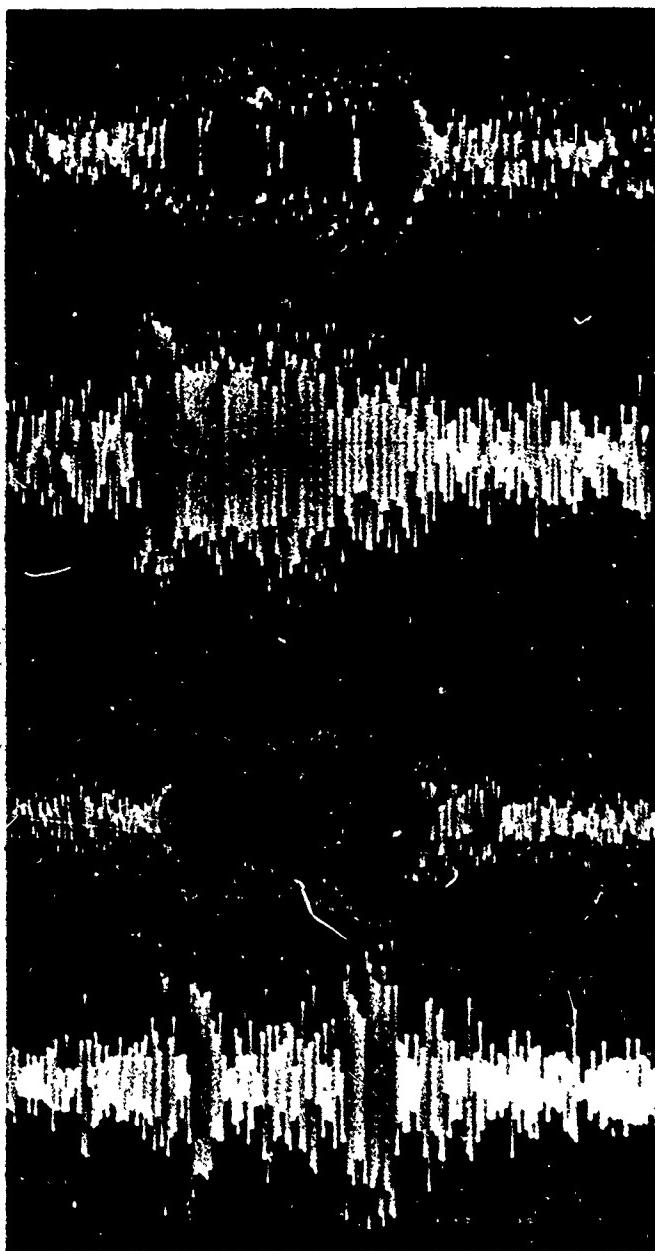
Figure 10   Oscilloscope photograph of 20-Hz signal phase-reversal

In this oscilloscope photograph of a 20-Hz sound from **Balaenoptera physalus**, finback whale, the signal from hydrophone B is superimposed on the signal from hydrophone C (the first arrival). The sweeps of the oscilloscope have been adjusted to present the two traces of the same sound approximately simultaneously. During the first portion, the traces are nearly 180° out of phase and then the signal on hydrophone B reverses its phase and the traces become nearly in-phase. The conditions are approximately those of Figure 8, the hydrophones are separated by nearly 30 m, and both are at a depth of 5 m in about 50 m of water. The whales are 1 km or more away.

NOT REPRODUCIBLE
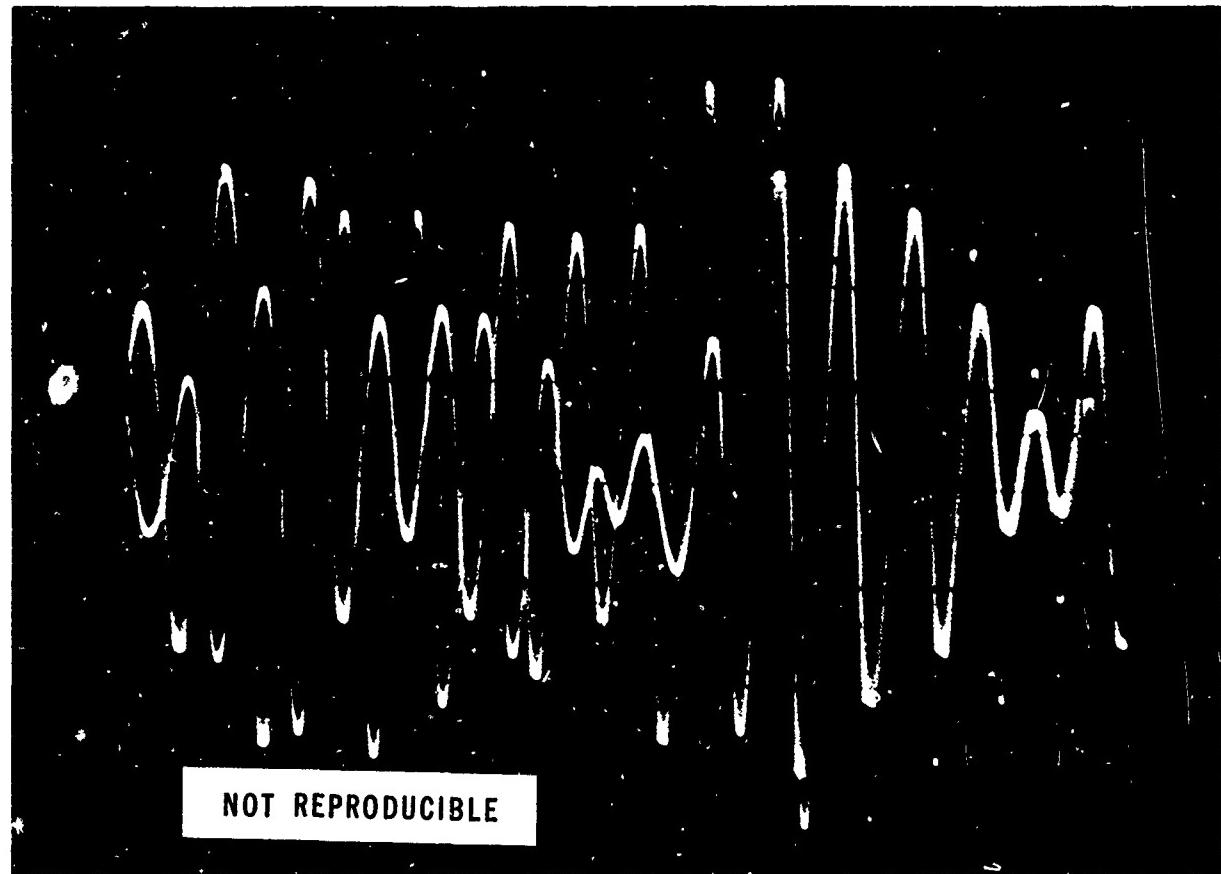
Figure 11   Spectrogram of <u>Eubalaena</u> sound

Two spectrographic pictures of the same two-second portion of underwater ambient are shown, first the signal from hydrophone C (the first arrival), and second from hydrophone B. The difference in level of the 400-Hz right whale (<u>Eubalaena</u>) sound received by the two hydrophones was approximately 17 dB more than it should have been for the observed distances. The hydrophones and their circuitry were essentially identical, and separated by about 30 m; both were at a depth of 5 m. The filter bandwidth of this analysis was 22.5 Hz.
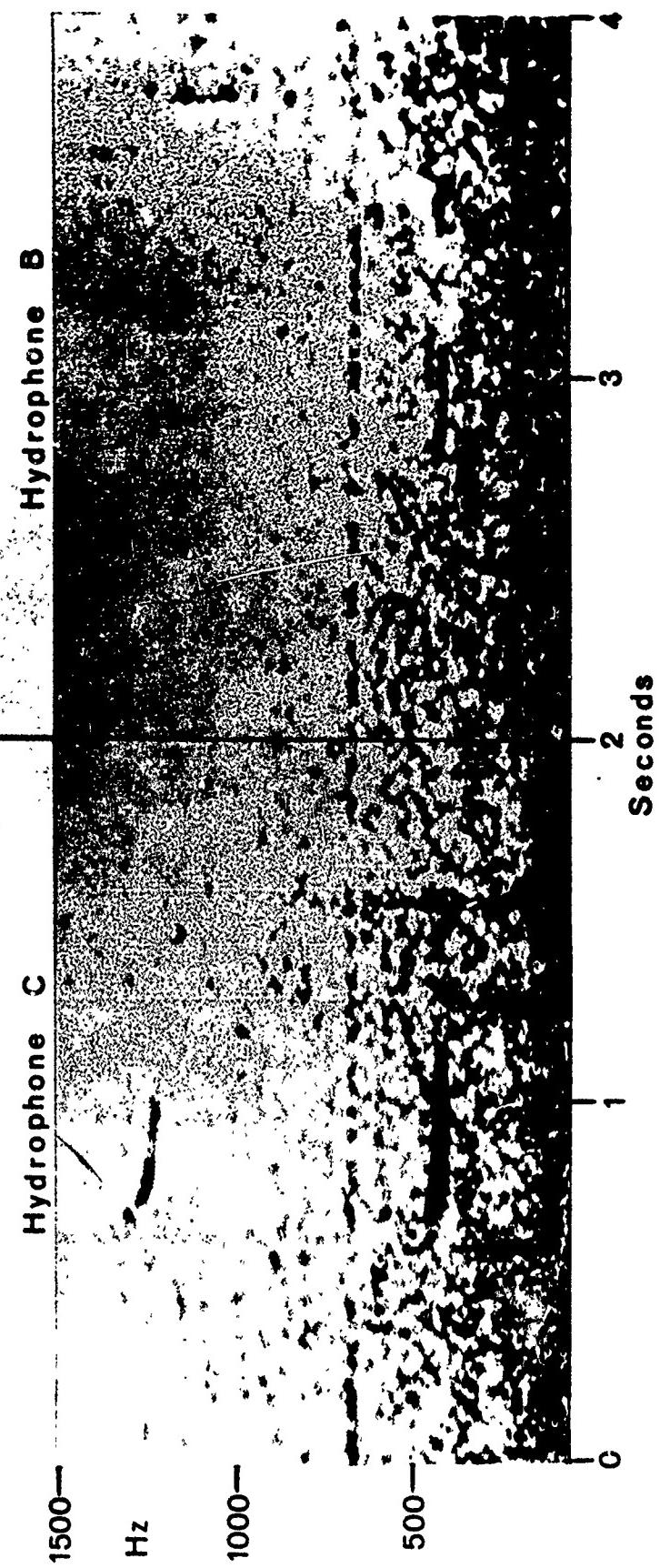
Figure 12   Spectrogram of <u>Lagenorhynchus</u> sounds

Squeals from several porpoises (<u>Lagenorhynchus</u> <u>albirostris</u>)
over-lap each other in this spectrograph and illustrate the difficulty
of isolating one sound for arrival-time-difference measurement.
The squeal that begins shortly after the 0.5 seconds marks was
measured successfully. The right whale sound of figure 11 begins
at about 0.7 seconds but is obscured in the low frequency noise in
this figure. The filter bandwidth of this analysis was 90 Hz.

TIME-SECONDS

kHz

16 —
12 —
8 —
4 —

0.5

0
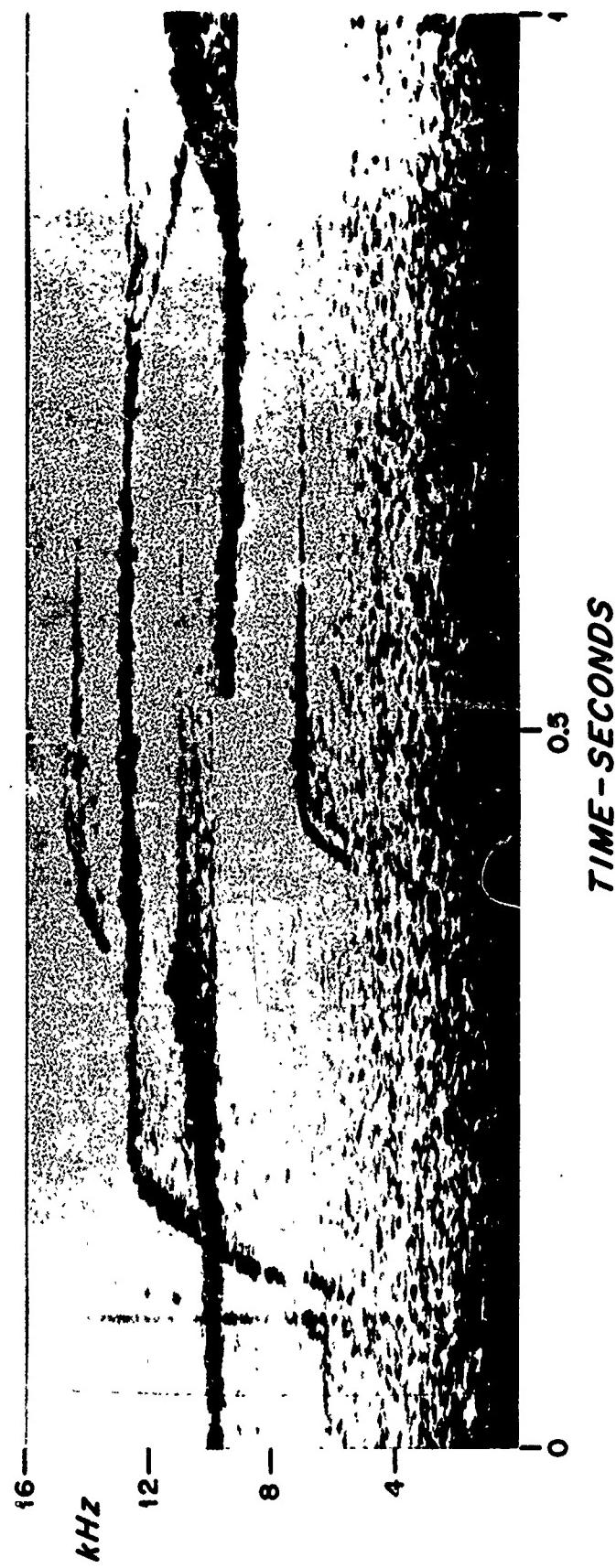
Figure 13   Computer printout for calibration

The calibration sheet produced by the computer program allows a check on the environmental data and measured dimensions that are used for the calculations. The calculated hydrophone positions are for that calibration alone. All succeeding (data) locations of sound sources are based on this information until a new calibration is introduced. Potential intensity differences are based only on relative distances to the hydrophones for each calibration pulse; the levels are compared to the first arrival and permit an assessment of the performance of the array systems.

CAPE COD BAY 70-14 128 CAL

KNOWN POSITIONS (ALL DISTANCES IN METERS):

BOW HYDROPHONE   X = 0.0   Y = 0.0   Z = 0.0
STERN HYDROPHONE X = .0   Z = .0
SIDE HYDROPHONE   X = +   Z = .0
CAL. PULSE #1   X = .0   Y = Y(2) Z = .5
CAL. PULSE #2   X = .0   Z = .5

DEPTH OF BOW HYDROPHONE = 5.0
DISTANCE BETWEEN PULSES = 34.007 (COMPUTED)
LENGTH OF LINE TO DEEP HYDROPHONE = 35.0
DEEP HYDROPHONE SUSPENDED FROM SIDE HYDROPHONE
SPEED OF SOUND = 1.525 METERS PER MILLISECOND

TIME DIFFERENCES   PULSE #1
          A   22.3   22.6   30.9
               PULSE #2
          B   26.3   28.1   35.5

HYDROPHONE POSITIONS:

BOW HYDROPHONE   X = .00   Y = .00   Z = .00
STRN HYDROPHONE   X = .00   Y =-34.00   Z = .00
SIDE HYDROPHONE   X = 33.78   Y =-27.17   Z = .00
DEEP HYDROPHONE   X = 30.63   Y =-28.25   Z =-34.84

INTENSITIES:

PULSE #1 -36.652   PULSE #2 .000
PULSE #1 .000   PULSE #2 -36.652
PULSE #1 -36.768   PULSE #2 -38.761
PULSE #1 -39.485   PULSE #2 -40.770

Figure 14   Computer printout for source location.

The calculated position of a sound source with the indicated

arrival-time differences, is given in x-y-z coordinates. In

addition, results of the error assessment, and potential intensity

differences based on relative distances to the calculated source

location, are printed out by the computer.   The two-dimensional

location is given when possible.

CAPE COD BAY 70-14 121 RIGHT WHALE

TIME DIFFERENCES

    C   18.17   8.   5.8

SOLUTIONS FOR DISTANCE =   62.85

X =   60.54 METERS   INTENSITY AT BOW RELATIVE TO FIRST ARRIVAL   -1.874
Y =   15.20 METERS   INTENSITY AT STRN RELATIVE TO FIRST ARRIVAL   -3.790
Z =   -7.32 METERS   INTENSITY AT SIDE RELATIVE TO FIRST ARRIVAL   .000
                         INTENSITY AT DEEP RELATIVE TO FIRST ARRIVAL   -1.398
                         INTENSITY AT SOURCE RELATIVE TO FIRST ARRIVAL 34.092

ERROR OF X =   .523E 01 ERROR OF Y =   .645E 01 ERROR OF Z =   .135E 01
ERROR OF X = -.523E 01 ERROR OF Y = -.645E 01 ERROR OF Z = -.135E 01

CALCULATIONS USING ONLY SURFACE DATA

SOLUTIONS FOR DISTANCE =   60.53

X =   58.85 METERS   INTENSITY AT BOW RELATIVE TO FIRST ARRIVAL   -1.955
Y =   14.14 METERS   INTENSITY AT STRN RELATIVE TO FIRST ARRIVAL   -3.937
                         INTENSITY AT SIDE RELATIVE TO FIRST ARRIVAL   .000
                         INTENSITY AT SOURCE RELATIVE TO FIRST ARRIVAL 33.684

APPENDIX: Program WHALOC

To find the location of a source from the difference

in the sound arrival time at four hydrophones in a

non-rigid array.

by

D. J. Ekstrand and M. Hunt

Information Processing Center

Woods Hole Oceanographic Institution

PROGRAM REPORT

NAME: WHALOC

TYPE: Main program

PURPOSE: To find the location of a sound source from the difference in
the arrival time of the sound at four hydrophones in a non-rigid
array.

MACHINE: XDS Sigma 7

SOURCE LANGUAGE: Fortran IV

PROGRAM CATEGORY: Mathematical

DESCRIPTION: Part I describes the various options available to the user.
Part II describes, when the coordinates of the hydrophones are
known, the determination of source position and possible error in
source position due to errors in arrival time differences.   Part III
describes the coordinate system and geometry of the hydrophone
array and calibration pulses which are assumed in order to deter-
mine the coordinates of the hydrophones.   Part IV describes the
calculations for intensity.

I.   Options available to the user

There are two different distinct logical procedures in the program.
The first is, to find the position of the four hydrophones from the
difference in the time of arrival at the four hydrophones of two
calibration pulses of known position.   The second is, to find the
location of a sound source from the differences in the arrival time
of the sound at the four hydrophones.   There are options available
for either procedure.

For the calibration, the options are:

A.   Read in the constants that describe the system, or use
those from the previous calibration.   Of course, on the
first calibration on any particular computer run, the
constants must be read in.

B.   Specify the geometry of the calibration pulses as one of
the following:

1.   Calibration pulse 1 is located at hydrophone A.   Pulse 2
is on the (imaginary) line joining hydrophones A and B.

2.   Calibration pulse 2 is located at hydrophone B.   Pulse 1
is on the (imaginary) line joining hydrophones A and B.

3. The calibration pulses are located off the line joining hydrophones A and B. The distance, P, between the two calibration pulses may be read or computed. If it is computed, the two pulses must be equidistant from the (imaginary) line between the A and B hydrophones. P is computed from the time differences between the A and B hydrophones and an estimate of the distance between them.

For locating the sound source, the options available are:

1. If the positions of the four hydrophones are known, it is not necessary to calibrate the system first. In this case, the hydrophone positions and the speed of sound are read in.

2. A calculation of the possible error in the location of the sound source may be done. This calculation is based on a possible error of ±1 millisecond in the time differences.

II. Determination of source position and possible error

We are given an array of four hydrophones located at the known positions

(X_i, Y_i, Z_i) , i= 1, 2, 3, 4.

A sound source is at the unknown position (W_1, W_2, W_3), and emits a sound at time 0. The time of arrival of the sound at hydrophone i is T_i.

We single out hydrophone 1 and write

$$T_i = T_1 + \tau_i \ (i = 1, 2, 3, 4) \tag{1}$$

Where τ_i is the difference between time of arrival at hydrophone i and time of arrival at hydrophone 1. The τ_i are known; in particular, $\tau_1 = 0$.

We assume the speed of sound is constant in space and time, and we call it C.

The distance between the sound source and hydrophone i is CT_i. Using the formula for the square of the distance between two points and substituting for T_i by equations (1), we have the following:

$$(W_1-X_i)^2 + (W_2-Y_i)^2 + (W_3-Z_i)^2 = C^2 (T_1+\tau_i)^2 \quad (i= 1, 2, 3, 4) \tag{2}$$

These are four simultaneous equations in the four unknowns W_1, W_2, W_3, T_1. The method of solution is given in Appendix p. 15.

We wish to determine the possible error in the calculated source position due to the error in measuring the arrival time differences.

We have calculated the coordinates W_1, W_2, W_3 of the source position as functions of the arrival time differences τ_2, τ_3, τ_4 and the arrival time T_1. We can write these functions as:

$$W_j = \alpha_j(T_1, \tau_2, \tau_3, \tau_4) \qquad (j = 1, 2, 3) \tag{3}$$

where α_j indicates the functional relationship, and we have indicated T_1 explicitly since T_1 also depends on τ_i (i = 2, 3, 4).

Suppose the error in τ_i is $\Delta\tau_i$ (i = 2, 3, 4). We assume the $\Delta\tau_i$ are small. Then we may approximate the error ΔW_j in W_j by the linear portion of the Taylor series for W_j (Reference 1; Vol. 1, p. 349 and Vol. 2, p. 68):

$$\Delta W_j = \sum_{i=2,3,4} \left[\frac{\partial\alpha_j}{\partial T_1} \frac{\partial T_1}{\partial\tau_i} + \frac{\partial\alpha_j}{\partial\tau_i} \right] \Delta\tau_i \qquad (j = 1, 2, 3) \tag{4}$$

The partial derivatives are evaluated at τ_2, τ_3, τ_4.

The details of the α_j and the partial derivatives are given on page 8.

III. Determination of hydrophone positions.

There are four hydrophones, called A, B, C, D, arranged in a three-dimensional array. We make the following assumptions about the geometry of the hydrophone array and the choice of coordinate system (see Figure 1):

1. Hydrophone B is taken as the origin.

2. The line connecting hydrophones A and B coincides with the y-axis; y increases from A to B.

3. The x-axis is perpendicular to the y-axis; the x-y plane is parallel to the sea surface.

4. The z-axis is perpendicular to the x-y plane; z increases up.

5. The x, y, z axes form a right-handed coordinate system.

6. The z-coordinates of hydrophones A and C are known.

7. The line to hydrophone D is attached to one of the hydrophones A, B, or C, or to a point whose coordinates are known. The line is of known length and is assumed to be straight.

(see Figure 3 of Array report)

The (x, y, z) coordinates of the hydrophones are

hydrophone	x, y, z
Bow B	X_1, Y_1, Z_1
Stern A	X_2, Y_2, Z_2
Side C	X_3, Y_3, Z_3
Deep D	X_4, Y_4, Z_4

In terms of the coordinates, our assumptions above give:

$$X_1 = Y_1 = Z_1 = X_2 = 0$$

Z_2 is known

Z_3 is known

There are two sound sources used to calculate the positions of the hydrophones. We make the following assumptions about the positions of the two calibration pulses (see Figure 1):

1. The y-coordinates of both sound sources are between Y_1 and Y_2.

2. The x-coordinates of both sources are known.

3. The z-coordinates of both sources are known.

The (x, y, z) coordinates of the calibration pulses are:

CALIBRATION PULSE	x, y, z
1	X_{c1}, Y_{c1}, Z_{c1}
2	X_{c2}, Y_{c2}, Z_{c2}

In terms of the coordinates, our assumptions above give:

$$Y_2 \leqslant Y_{c1} < Y_{c2} \leqslant Y_1$$

X_{c1}, X_{c2} are known

Z_{c1}, Z_{c2} are known

The sound source k (k= 1, 2) emits a sound at time 0. The time of arrival of the sound at hydrophone i is T_{ik} (i = 1, 2, 3, 4).

We single out hydrophone 1 and write

$$T_{ik} = T_{1k} + \tau_{ik} \quad (i=1, 2, 3, 4, k=1, 2) \tag{5}$$

where τ_{ik} is the difference between time of arrival of pulse k at hydrophone i and time of arrival at hydrophone 1. The τ_{ik} are known; in particular, $\tau_{11} = \tau_{12} = 0$.

We assume the speed of sound is constant in space and time, and we call it C.

The distance between the sound source k and hydrophone i is CT_{ik}. Using the formula for the square of the distance between two points and substituting for T_{ik} by equation (5), we have the following equations:

Distances between hydrophone B and pulses 1 and 2

$$X_{c1}{}^2 + Y_{c1}{}^2 + Z_{c1}{}^2 = C^2 T_{11}{}^2 \tag{6}$$

$$X_{c2}{}^2 + Y_{c2}{}^2 + Z_{c2}{}^2 = C^2 T_{12}{}^2 \tag{7}$$

Distances between hydrophone A and pulses 1 and 2

$$(X_{c1}-X_2)^2 + (Y_{c1}-Y_2)^2 + (Z_{c1}-Z_2)^2 = C^2(T_{11}+\tau_{21})^2 \tag{8}$$

$$(X_{c2}-X_2)^2 + (Y_{c2}-Y_2)^2 + (Z_{c2}-Z_2)^2 = C^2(T_{12}+\tau_{22})^2 \tag{9}$$

Distance between calibration pulses

$$(X_{c1}-X_{c2})^2 + (Y_{c1}-Y_{c2})^2 + (Z_{c1}-Z_{c2})^2 = p^2 \tag{10}$$

For calibration <u>option 1</u>, we have pulse 1 located at hydrophone A
and pulse 2 on the line joining A and B. In terms of the coordinates,

$$X_{c1} = X_{c2} = 0$$

$$Y_{c1} = Y_2$$

$$Z_{c1} = Z_2$$

$$T_{11} = -\tau_{21}$$
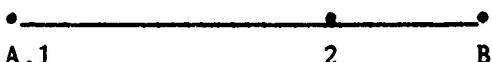
Equations (6) to (10) become

$$Y_2{}^2 + Z_{c1}{}^2 = C^2 \tau_{21}{}^2$$

$$Y_{c2}{}^2 + Z_{c2}{}^2 = C^2 T_{12}{}^2$$

$$(Y_{c2}-Y_2)^2 + (Z_{c2}-Z_2)^2 = C^2(T_{12}+\tau_{21})^2 = p^2$$

where $p = \frac{C}{2}(\tau_{22}-\tau_{21})$ is the distance between the calibration pulses

(see Appendix p. 20.)

These are three equations in the three unknowns Y_2, Y_{c2}, T_{12}. The are
solved directly.

For calibration <u>option 2</u>, we have pulse 2 located at hydrophone B and
pulse 1 on the line joining A and B. In terms of the coordinates,

$$X_{c1} = X_{c2} = 0$$

$$Y_{c2} = Y_1 = 0$$

$$Z_{c2} = Z_1 = 0$$

$$T_{12} = 0$$
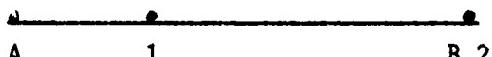
Equations (6) to (10) become

$$Y_{c1}^2 + Z_{c1}^2 = C^2 T_{11}^2 = p^2$$

$$(Y_{c1} - Y_2)^2 + (Z_{c1} - Z_2)^2 = C^2(T_{11} + \tau_{21})^2$$

$$Y_2^2 + Z_2^2 = C^2 \tau_{22}^2$$

where $P = \frac{C}{2}(\tau_{22} - \tau_{21})$ is the distance between the calibration pulses
(see Appendix p. 20.)

These are three equations in the three unknowns Y_2, Y_{c1}, T_{11}. They are
solved directly.

For calibration option 3, we assume that the distance P between the
calibration pulses is known. An alternative to this assumption, where
P is calculated from an estimate of the distance between hydrophones
A and B, is given at the end of this section.

Equations (6) to (10) are five simultaneous equations in the five
unknowns Y_{c1}, Y_{c2}, Y_2, T_{11}, T_{12}. We solve them by the Newton-Raphson
method (Reference 2, p. 213). The method is described in Appendix B.

Now we know the coordinates of the calibration pulse positions, the
coordinates of hydrophone A, and the arrival times of the pulses at
hydrophone B. The calculations for the coordinates of hydrophones C
and D are the same for options 1, 2, 3.

The distances between hydrophone C and pulses 1 and 2 are:

$$(X_{c1} - X_3)^2 + (Y_{c1} - Y_3)^2 + (Z_{c1} - Z_3)^2 = C^2(T_{11} + \tau_{31})^2 \tag{11}$$

$$(X_{c2} - X_3)^2 + (Y_{c2} - Y_3)^2 + (Z_{c2} - Z_3)^2 = C^2(T_{12} + \tau_{32})^2 \tag{12}$$

Equations (11) and (12) are two simultaneous equations in the two
unknowns X_3, Y_3. We solve them by the Newton-Raphson method, described
on page 15.

The distances between hydrophone D and pulses 1 and 2 are:

$$(X_{c1} - X_4)^2 + (Y_{c1} - Y_4)^2 + (Z_{c1} - Z_4)^2 = C^2(T_{11} + \tau_{41})^2 \tag{13}$$

$$(X_{c2} - X_4)^2 + (Y_{c2} - Y_4)^2 + (Z_{c2} - Z_4)^2 = C^2(T_{12} + \tau_{42})^2 \tag{14}$$

Hydrophone D is attached to point (X_L, Y_L, Z_L) by a line of length L:

$$(X_L-X_4)^2 + (Y_L-Y_4)^2 + (Z_L-Z_4)^2 = L^2 \tag{15}$$

For options 1 and 2, the point (X_L, Y_L, Z_L) may not be on the line joining hydrophones A and B.

Equations (13) to (15) are three simultaneous equations in the three unknowns X_4, Y_4, Z_4. We solve them by the Newton-Raphson method, described on Appendix p. 18.

This completes the determination of the positions of the four hydrophones, based on information from the two calibration pulses.

An alternative to the assumption that the distance P between the two calibration pulses is known is the following set of assumptions:

1. Hydrophones A and B and pulses 1 and 2 are in the same plane; for example, their z-coordinates are equal,
    $Z_2 = Z_{c1} = Z_{c2} = 0$.

2. The x-coordinates of pulses 1 and 2 are equal; call
    $d = X_{c1} = X_{c2}$.

3. The distance between hydrophones A and B is known approximately.

An estimated P_{est} is found. A new estimate P_{cest} is calculated. Then Then the distance P between the calibration pulses is calculated as:

$$P = P_{est} + P_{est} (P_{est} - P_{cest})/P_{cest} \tag{16}$$

Equation (16) is derived in Appendix p. 20.

IV. Calculation of intensities

Relative sound intensities at all hydrophones are found for both the calibration pulses and the sound source. These computations are all based on the assumption that the sound source is in fact at the computed position. The intensities are computed by the equation:

$$I = -20 \log \frac{D_1}{D_0}$$

where I is the intensity, D_1 is the computed distance from the sound source to the hydrophone, and D_0 is the computed distance from the sound source to the hydrophone where the sound arrived first.

INPUT: Input is all from cards. There are two different card sequences, one for calibration, and one for data. The type of sequence is indicated by the first column of card 1, which is an identification and option card used by both sequences. As many sets of each sequence as desired may be used, the only restriction being that a data set may not be processed unless the hydrophone positions are known.

The layout of the various cards is as follows:

Card 1 (either calibration or data)

column	name	definition
1	KIND	'1' means calibration set follows. '0' means data set follows. '9' means end of run.
2	IRD	'1' for calibration set means use previous hydrophone constant. '1' for data set means skip error computation. Otherwise leave blank.
3-78	ID	Alphanumeric identification.
80	NOPT	Used for data set only. '1' means read in hydrophone positions. Otherwise leave blank.

The following cards are for calibration.

If column 2 of Card 1 has a '1', omit cards 2C through 5C.

Card 2C

column	name	definition
2-10	SIDE	+1.0 if x coordinate of C hydrophone is positive. -1.0 if x coordinate of C hydrophone is negative.
11-20	$Z(2)$	Depth of A hydrophone relative to B hydrophone.
21-30	$Z(3)$	Depth of C hydrophone relative to B hydrophone.

Card 3C

column	name	definition
2-10	XC1	x coordinate of calibration pulse # 1.
11-20	XC2	x coordinate of pulse #2.
21-30	ZC1	z coordinate of pulse #1.
31-40	ZC2	z coordinate of pulse #2.

Card 4C

column	name	definition
2	NICK	'A' means D hydrophone suspended from A hydrophone. 'B' means suspended from B. 'C' means suspended from C. 'D' means coordinates of end of line to hydrophone D will be read.
6-15	AL	Length of line to hydrophone D.
16-25	C	Speed of sound (in meters per millisecond).
26-35	DPTH	Depth of B hydrophone
36-45 46-55 56-65	XL YL ZL	Coordinates of end of line to D hydrophone (if 'D' is punched in column 2).

Card 5C

column	name	definition
2	JOPT	Used to specify hydrophone layout. '1' means pulse #1 is with 'A' hydrophone. '2' means pusle #2 is with 'B' hydrophone. '3' means pulses are not on line between A and B hydrophones.
6-15	P	For option '3' above only Distance between calibration pulses. If left blank, P will be computed.
16-25	ABEST	For option '3' above only. Estimate of distance between A and B hydrophones. Must be specified if P is not.
26-35	PERC	Convergence limit (punch decimal). If left blank, program sets PERC= .01.
36-40	ITER	Number of iterations (right-justify). If left blank, program sets ITER=20.

Card 6C

column	name	definition
2	IT	Hydrophone where sound from pulse #1 arrived first (A, B, C, or D).
6-15 16-25 26-35	TD(1) TD(2) TD(3)	Arrival time differences from hydrophone specified in column 2. Must be in alphabetical order.

Appendix -11-

<u>Card 7C</u>

Similar to Card 6C but for pulse # 2

This is the end of the calibration cards.

The data cards are as follows:

Cards 2D through 5D are included <u>only</u> when by-passing the calibration procedure and reading in the hydrophone locations.

<u>Card 2D</u>

column	name	definition
2-10	C	Speed of sound (in meters per millisecond).

<u>Card 3D</u>

column	name	definition
2-10	X(2)	
11-20	Y(2)	Coordinates of hydrophone A.
21-30	Z(2)	

<u>Card 4D</u>

column	name	definition
2-10	X(3)	
11-20	Y(3)	Coordinates of C hydrophone
21-30	Z(3)	

<u>Card 5D</u>

column	name	definition
2-10	X(4)	
11-20	Y(4)	Coordinates of D hydrophone.
21-30	Z(4)	

<u>Card 6D</u>

column	name	definition
2	IT	Hydrophone where sound arrived first (A, B, C, or D).
6-15	TD(1)	Arrival time differences from hydrophone specified
16-25	TD(2)	in column 2. Must be in alphabetical order.
26-35	TD(3)	

This is the end of the data set. The program will return to read Card 1. A '9' in column 1 of Card 1 means end of all data.

A few words about the proper form for cards 6C, 7C, and 6D. If, for instance, the sound arrived first at hydrophone C, 'C' would be punched in column 2, the difference between the time to A and the time to C in columns 6-15, between B and C in columns 16-25, and between D and C in columns 26-35.

It will be noticed that card column 1 is blank in all but Card 1.

OUTPUT: There is one page of output on the line printer for each calibration set of data and one page for each data set. In either case, the identification is printed at the top of the page.

For the calibration, the calibration constants are printed, then the time differences for the two calibration pulses, and finally the computed hydrophone locations. For a data set, output includes first, the input time differences. Then, the computed value of the time to the bow hydrophone and the computed location of the sound source. If so specified in the input option, an estimate of the errors in the source location is computed and printed. For any one data set, there may be two solutions. A solution for which the time is negative is, of course, meaningless. If both solutions of time are positive, the user must determine the correct solution.

USAGE: The job deck should be set up as follows:

```
!JOB X, Y
!OLAY (UNSAT, (3), (F4LIB))
        binary deck of WHALOC
!RUN
! DATA
        data cards as described
```

RESTRICTIONS:

1.   All input data should be as accurate as possible.

2.   The program assumes a constant sound velocity.

3.   Calibration option 1 and 2 give the best results.

Restrictions 4 and 5 apply to calibration option # 3.

4.   Under certain geometries of calibration pulse positions, a very small error in the value of P can cause gross errors in the calculated locations of the hydrophones. This can be avoided by positioning the calibration pulses so that the ratio $D/AB \geq .20$, where AB is the distance between the A and B hydrophones, and D is the distance of the pulses from the line between the A and B hydrophones. The pulses should also be located on the opposite side of the line AB from the C hydrophone.

In cases where the ratio D/AB is as small as .04, an error in P of less than .1% leads to intolerable errors in all of the results. When the ratio D/AB is as large as .3, and error of 8% in the value of P will give fairly good results, and if D/AB equals .6, a 10% error in P gives good results.

5. One of the methods for computing P depends on an estimate of the distance AB. It should be borne in mind that the Y-coordinate of A hydrophone (which equals -AB) is computed from P. If P is first computed from AB, the computed Y-coordinate of the A hydrophone will be very close to the estimate. Optimum results will be achieved when the distance between the calibration pulses is known.

STORAGE REQUIREMENTS:

The program, including all subsidiary routines, requires 9216_{10} locations.

SUBPROGRAMS REQUIRED: Subroutine DSLEQ

OPERATIONAL ENVIRONMENT: Standard

OPERATIONAL CHARACTERISTICS:

The program alternately reads a few cards, and computes and prints out answers, until the end of the data is reached.

TIMING: Approximately .1 minute for one calibration and two data sets.

ERRORS & DIAGNOSTICS:

1. If an attempt is made to calibrate without having read the calibration constants, the program prints:

'WHERE ARE HYDROPHONE CONSTANTS'

and stops.

2. If an attempt is made to run a data set without having calibrated the system or read in the hydrophone positions, the program prints:

'MUST CALIBRATE FIRST'

and continues reading cards until a calibration is found.

3. If there is not an 'A', 'B', 'C', or 'D' in column 2 of Card 4C, the program prints:

'DEEP HYDROPHONE SUSPENSION INCORRECTLY SPECIFIED'

and stops.

4. If there is not an 'A', 'B', 'C', or 'D' in column 2 of
Card 6C, 7C, or 6D, the program prints:

'TIME DIFFERENCES INCORRECTLY SPECIFIED'

and stops.

5. When one of the three calibration procedures fails to converge,
the program prints:

'NO CONVERGENCE ON CALIB. #____. CONVERGED TO

INCORRECT VALUES ____ TIMES. DID NOT CONVERGE

AFTER ____ ITERATIONS ____ TIMES. ESTIMATES WERE

____ AND ____.'

where the appropriate numbers are substituted for the blanks. The
program will continue to read cards until it comes to the next calibration.

6. When the time differences given in a data set do not produce a solution,
the program prints:

'NO SOLUTION'

and continues with the next data set or calibration.

Both error diagnostics 5 and 6 indicate that the information supplied des-
cribes a physically impossible situation. Data should be checked carefully.

7. If one of the matrices used in the calibration procedure is
singular, the program will print:

'SINGULAR MATRIX ON CALIB. #_____.'

The program will continue to read cards until it comes to the
next calibration.

PROGRAMMER:
Program written by Mary Hunt. Mathematical analysis by
Donna Ekstrand.

ORIGINATOR: William Watkins

DATE: February, 1971

REFERENCES:
1. R. Courant, "Differential and Integral Calculus", vols. 1 and
2, Interscience Publishers, Inc., New York, 1937.

2. J. B. Scarborough, "Numerical Mathematical Analysis", Johns
Hopkins Press, Baltimore, 1962.

Source position and possible error.

We have the four simultaneous equations

$$(W_1-X_i)^2 + (W_2-Y_i)^2 + (W_3-Z_i)^2 = C^2(T_1+\tau_i)^2 \qquad (i=1,2,3,4) \qquad \text{(A-1)}$$

in the four unknowns W_1, W_2, W_3, T_1.

We subtract the first equation from the other three equations, substitute the known values $X_1 = Y_1 = Z_1 = 0$, and get three equations linear in the four unknowns:

$$2X_iW_1 + 2Y_iW_2 + 2Z_iW_3 = -2C^2\tau_iT_1-C^2\tau_i^2-R_i \qquad (i=2,3,4) \qquad \text{(A-2)}$$

where $R_i = -(X_i^2 + Y_i^2 + Z_i^2)$.

We solve these three equations for W_1, W_2, W_3 as linear expressions of T_1, written as

$$W_j = f_jT_1 + g_j \qquad\qquad (j=1,2,3) \qquad \text{(A-3)}$$

$$f_j = S_{2j}\tau_2 + S_{3j}\tau_3 + S_{4j}\tau_4 \qquad\qquad (j=1,2,3) \qquad \text{(A-4)}$$

$$g_j = U_j + \tfrac{1}{2}\left[S_{2j}\tau_2^2 + S_{3j}\tau_3^2 + S_{4j}\tau_4^2\right] \qquad (j=1,2,3) \qquad \text{(A-5)}$$

The S_{ij} and U_j are constants; the expressions for these, in terms of the X_i, Y_i, Z_i ($i=2,3,4$) and C, are listed at the end of the appendix.

The f_j and g_j are functions only of the τ_i.

We substitute equations (A-3) into the first of equations (A-1), and solve for T_1. We have a quadratic equation for T_1:

$$aT_1^2 + 2bT_1 + d = 0 \qquad\qquad \text{(A-6)}$$

The a, b, d are functions of the τ_i:

$a = f_1{}^2 + f_2{}^2 + f_3{}^2 - C^2$

$b = f_1 g_i + f_2 g_2 + f_3 g_3$

$d = g_1{}^2 + g_2{}^2 + g_3{}^2$

Then we solve equation (6) for T_1:

$$T_1 = (-b \pm \sqrt{b^2 - ad})/a \tag{A-7}$$

Since T_1 is the time for the sound to reach hydrophone B, it should be positive. From here on, we work with a positive T_1, if one exists.

We substitute T_1, as given by equation (A-7), into equations (A-3). Thus, we have calculated W_1, W_2, W_3, the coordinates of the sound source.

It remains to find the possible error in the W_j, due to a possible error in the τ_i (i = 2, 3, 4). We have, by equations (A-3),

$$W_j = \alpha_j\ (T_1, \tau_2, \tau_3, \tau_4) = f_j\ (\tau_2, \tau_3, \tau_4)\ T_1 + g_j\ (\tau_2, \tau_3, \tau_4)$$
$$(j = 1, 2, 3) \tag{A-8}$$

By equations (4) of the Description, we know

$$\Delta W_j = \sum_{i=2,3,4} \left[\frac{\partial \alpha_j}{\partial T_1} \frac{\partial T_1}{\partial \tau_i} + \frac{\partial \alpha_j}{\partial \tau_i} \right]\ \Delta \tau_i \qquad (j = 1, 2, 3) \tag{A-9}$$

We must write expressions for the various partial derivatives. From equations (A-4) and (A-5),

$$\frac{\partial f_j}{\partial \tau_i} = S_{ij} \quad \text{and} \quad \frac{\partial g_j}{\partial \tau_i} = S_{ij}\tau_i, \quad \text{so}$$

$$\frac{\partial \alpha_j}{\partial \tau_i} = \frac{\partial f_j}{\partial \tau_i}\ T_1 + \frac{\partial g_j}{\partial \tau_i} = S_{ij}\ T_1 + S_{ij}\ \tau_i$$

There remains the first term in the square brackets of equation (A-9).

$$\frac{\partial \alpha_j}{\partial T_1} = f_j$$

$$\frac{\partial a}{\partial \tau_i} = 2 \sum_{j=1,2,3} f_j \frac{\partial f_j}{\partial \tau_i} = 2 \sum_{j=1,2,3} f_j S_{ij}$$

$$\frac{\partial b}{\partial \tau_i} = \sum_{j=1,2,3} (\frac{\partial f_j}{\partial \tau i} g_j + f_j \frac{\partial g_j}{\partial \tau_i}) = \sum_{j=1,2,3} (S_{ij} g_j + f_j S_{ij} \tau_i)$$

$$\frac{\partial d}{\partial \tau_i} = 2 \sum_{j=1,2,3} g_j \frac{\partial g_j}{\partial \tau_i} = 2 \sum_{j=1,2,3} g_j S_{ij} \tau_i$$

Finally, we have

$$\frac{\partial T_1}{\partial \tau_i} = \frac{1}{a^2} \left[a(- \frac{\partial b}{\partial \tau_i} \pm \frac{1}{2}(b^2-ad)^{-\frac{1}{2}} (2b \frac{\partial b}{\partial \tau_i} - \frac{\partial a}{\partial \tau_i} d - a \frac{\partial d}{\partial \tau_i})) - (-b \pm (b^2-ad)^{\frac{1}{2}}) \frac{\partial a}{\partial \tau_i} \right]$$

We list the expressions for the constants

S_{ij} and U_j (j = 1, 2, 3 ; i = 2, 3, 4):

$$\det = 8(X_2 Y_3 Z_4 + X_3 Y_4 Z_2 + X_4 Y_2 Z_3 - X_2 Y_4 Z_3 - X_3 Y_2 Z_4 - X_4 Y_3 Z_2)$$

$$S_{21} = -8(Y_3 Z_4 - Y_4 Z_3) \, C^2/\det$$

$$S_{31} = 8(Y_2 Z_4 - Y_4 Z_2) \, C^2/\det$$

$$S_{41} = -8(Y_2 Z_3 - Y_3 Z_2) \, C^2/\det$$

$$S_{22} = 8(X_3 Z_4 - X_4 Z_3) \, C^2/\det$$

$$S_{32} = -8(X_2 Z_4 - X_4 Z_2) \, C^2/\det$$

$$S_{42} = 8(X_2 Z_3 - X_3 Z_2) \, C^2/\det$$

$$S_{23} = -8(X_3 Y_4 - X_4 Y_3) \, C^2/\det$$

$$S_{33} = 8(X_2 Y_4 - X_4 Y_2) \, C^2/\det$$

$$S_{43} = -8(X_2 Y_3 - X_3 Y_2) \, C^2/\det$$

$$U_1 = -4 \left[R_2(Y_3 Z_4 - Y_4 Z_3) - R_3(Y_2 Z_4 - Y_4 Z_2) + R_4(Y_2 Z_3 - Y_3 Z_2) \right] /\det$$

$$U_2 = -4 \left[-R_2(X_3 Z_4 - X_4 Z_3) + R_3(X_2 Z_4 - X_4 Z_2) -R_4(X_2 Z_3 -X_3 Z_2) \right] /\det$$

$$U_3 = -4 \left[R_2(X_3 Y_4 - X_4 Y_3) - R_3(X_2 Y_4 - X_4 Y_2) + R_4(X_2 Y_3 - X_3 Y_2) \right] /\det$$

Application of the Newton-Raphson method for solving simultaneous equations.

See J. B. Scarborough, p. 213-215 (Reference 1). The following discussion will be for two equations in two unknowns. Suppose the equations are:

$$\phi(X,Y) = 0$$

$$\psi(X,Y) = 0$$

Let X_0, Y_0 be approximate values of a pair of roots, and let h,k be corrections, so that

$$X = X_0 + h$$

$$Y = Y_0 + k$$

The equations become

$$\phi(X_0+h, \ Y_0+k) = 0$$

$$\psi(X_0+h, \ Y_0+k) = 0$$

We expand ϕ and ψ by Taylor's theorem, neglect terms in h and k of degree greater than one. The equations become

$$\phi(X_0,Y_0) + h\left(\frac{\partial\phi}{\partial X}\right)_0 + k\left(\frac{\partial\phi}{\partial Y}\right)_0 = 0$$

$$\psi(X_0,Y_0) + h\left(\frac{\partial\psi}{\partial X}\right)_0 + k\left(\frac{\partial\psi}{\partial Y}\right)_0 = 0$$

These equations are linear in h and k, and can be easily solved by a Gauss reduction routine.

Additional corrections are found in the same way until the corrections are small.

The initial estimate for the solution should be good. It is not guaranteed that the method will converge to the desired solution.

The equations for the corrections h can be written in matrix form:

$$\phi + Mh = 0$$

where the original equations are $\phi = 0$

ϕ is the vector of functions

h is the vector of corrections to the estimate vector X_0

M is the matrix of partial derivatives $m_{ij} = \left(\dfrac{\partial \phi_i}{\partial X_j}\right)_0$

The method is used three times in determining the hydrophone positions. In each case, the ϕ_i are just the left-hand side of the equations, after everything is taken over to the left side, so the right side is 0.

For position of hydrophone A, calibration pulses, and times of arrival to hydrophone B, when using option #3, the matrix M is:

$2Y_{c1}$	0	0	$-2C^2 T_{11}$	0
0	$2Y_{c2}$	0	0	$-2C^2 T_{12}$
$2(Y_{c1}-Y_2)$	0	$-2(Y_{c1}-Y_2)$	$-2C^2(T_{11}+\tau_{21})$	0
0	$2(Y_{c2}-Y_2)$	$-2(Y_{c2}-Y_2)$	0	$-2C^2(T_{12}+\tau_{22})$
$2(Y_{c1}-Y_{c2})$	$-2(Y_{c1}-Y_{c2})$	0	0	0

For position of hydrophone C, the matrix M is:

$-2(X_{c1}-X_3)$	$-2(Y_{c1}-Y_3)$
$-2(X_{c2}-X_3)$	$-2(Y_{c2}-Y_3)$

For position of hydrophone D, the matrix M is:

$$-2(X_{c1}-X_4) \qquad -2(Y_{c1}-Y_4) \qquad -2(Z_{c1}-Z_4)$$

$$-2(X_{c2}-X_4) \qquad -2(Y_{c2}-Y_4) \qquad -2(Z_{c2}-Z_4)$$

$$-2(X_L-X_4) \qquad -2(Y_L-Y_4) \qquad -2(Z_L-Z_4)$$

Appendix C. Approximation of distance between calibration pulses.

We assume the following:

1. Hydrophones A, B and pulses 1, 2 are in the same plane.

2. The X-coordinates of pulses 1, 2 are equal and known; call this distance d.

3. The distance between hydrophones A and B is known approximately; call this distance d_{AB}.

We want to calculate the distance P between pulses 1 and 2. See Figure C-1.
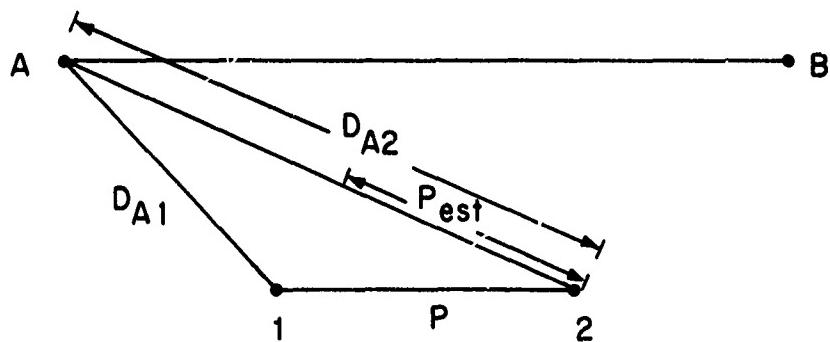
Figure C-1.

First, we estimate P by equation (C-1). This calculation is exact when pulses 1 and 2 are on the line connecting hydrophones A and B. (See Figure C-2)

D_{A1} = distance between hydrophone A and pulse 1.

D_{A2} = distance between hydrophone A and pulse 2.

$$2 P_{est} = (D_{A2}-D_{A1}) + (D_{B1}-D_{B2})$$
$$= (D_{A2}-D_{B2}) - (D_{A1}-D_{B1})$$
$$= C\tau_{22} - C\tau_{21}$$

So that $P_{est} = \frac{1}{2}C(\tau_{22}-\tau_{21})$ (C-1)
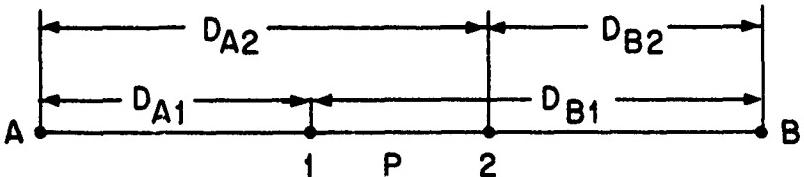
Figure C-2.

This first estimate P_{est} can also be derived as follows. Assume pulses 1 and 2 are symmetrically placed with respect to hydrophones A and B. Then $P_{est} = D_{A2}-D_{A1}$ (see Figure C-1). Note that P_{est} is smaller than P.

Now we calculate d_{A1} and d_{A2}, assuming the calibration pulses are distance P_{est} apart and are symmetrically placed with respect to hydrophones A and B (see Figure C-3). By right triangles A1E and A2F, we have

$$d_{A1} = \left[(\tfrac{1}{2}d_{AB} - \tfrac{1}{2}P_{est})^2 - d^2 \right]^{\tfrac{1}{2}}$$

$$d_{A2} = \left[(\tfrac{1}{2}d_{AB} + \tfrac{1}{2}P_{est})^2 - d^2 \right]^{\tfrac{1}{2}}$$
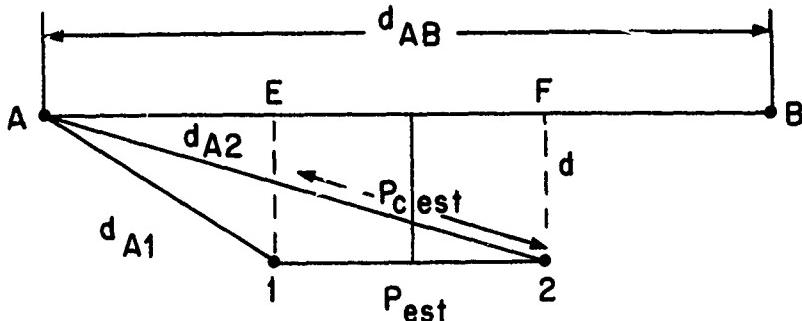

Figure C-3.

A new estimate for P is $P_{c\ est} = d_{A2} - d_{A1}$. Note that $P_{c\ est}$ is smaller than P_{est}.

We now assume that the ratio of the calculated estimate to the actual value is the same in the two cases. That is,

$$\frac{P_{c\ est}}{P_{est}} = \frac{P_{est}}{P}$$

Then, we have

$$P = P_{est}^{2}/P_{c\ est}$$

or $\quad P = P_{est} + P_{est} (P_{est} - P_{c\ est})/P_{c\ est}$